Opportunistic Relaying for Space-Time Coded Cooperation with Multiple Antennas Terminals

Behrouz Maham[†‡] and Are Hjørungnes[†]

[†]UNIK – University Graduate Center, University of Oslo, Norway

[‡]Department of Electrical Engineering, Stanford University, Stanford, USA

Email: bmaham@stanford.edu, arehj@unik.no

***Abstract*— We consider a wireless relay network with multiple antennas terminals over Rayleigh fading channels, and apply distributed space-time coding (DSTC) in *amplify-and-forward* (A&F) mode. It turns out that, combined with power allocation in the relays, A&F DSTC results in an opportunistic relaying scheme, in which the *best* relay is selected to retransmit the source's space-time coded signal. Next, assuming M-PSK or M-QAM modulations, we analyze the performance of the cooperative diversity wireless networks using A&F opportunistic relaying with the multiple-antennas source and destination. We first derive the probability density function (PDF) of the received SNR at the destination. Then, the PDF is used to determine the symbol error rate in Rayleigh fading channels. Then, we derived closed-form approximations for SER in high SNR scenario, from which we find the diversity order of system $R\min\{N_s, N_d\}$, where R, N_s, and N_d are the number of the relays, the source antennas, and the destination antennas, respectively. Simulation results show that the proposed system obtain 2 dB gain in SNR over DSTC for BER 10^{-5}, when $R = 2$, $N_s = 2$, $N_d = 2$.**

I. Introduction

In [1], a cooperative strategy was proposed which achieves a diversity factor of R in a R-relay wireless network, using the so-called distributed space-time codes (DSTC). A two-phase protocol is used for this strategy. In the first phase, the transmitter sends the information signal to the relays and in the second phase, relays send information to the receiver. The signal sent by every relay in the second phase is designed as a linear function of its received signal. It was shown that the relays can generate a linear space-time codeword at the receiver, as in a multiple antenna system, although they only cooperate distributively. This method does not require decoding at the relays and for high SNR it achieves the optimal diversity factor [1]. Although distributed space-time coding does not need instantaneous channel information in the relays, it requires full channel information at the receiver, i.e., both the channels from the transmitter to relays and the channels from relays to the receiver, need to be known at the receiver. This requires that training symbols are sent from both the transmitter and relays. Recently, the design of practical DSTC in amplify-and-forward (A&F) mode, that lead to reliable communication in wireless relay networks, has been presented in [2], [3], and [4].

This work was supported by the Research Council of Norway through the project 176773/S10 entitled "Optimized Heterogeneous Multiuser MIMO Networks – OptiMO".

Distributed space-time coding in A&F mode was generalized to networks with multiple-antenna nodes in [5]. It is shown that in a wireless network with N_s antennas at the transmit node, N_d antennas at the receive node, and a total of R single relay nodes, the diversity order of $R\min\{N_s, N_d\}$ is achievable [5], [6].

In [7], an opportunistic relaying scheme is introduced. According to opportunistic relaying, a single relay among a set of R relay nodes is selected, depending on which relay provides for the *best* end-to-end path between source and destination. In this paper, we propose a decision metric for opportunistic relaying based on maximizing the received instantaneous SNR at the destination in amplify-and-forward (A&F) mode, when multiple-antenna source and destination is used and statistical CSI of the source-relay channel is available at the relay.

We make the following main contributions in this paper:

- We show that the DSTC based on [1] in a rely network with the multiple-antennas source and destination leads to a new opportunistic relaying, when maximum instantaneous SNR based power allocation is used.
- The performance of the A&F opportunistic relaying with space-time coded source is analyzed. More specifically, we derive the average symbol error rate (SER) of opportunistic relaying with M-PSK or M-QAM modulations in a Rayleigh-fading channels. Furthermore, the probability density function (PDF) of the received SNR at the destination is obtained.
- For sufficiently high SNR, a simple closed-form average SER expression is derived for A&F opportunistic relaying links with multiple cooperating branches and multiple antennas source/destination. Based on the proposed approximated SER expression, it is shown that the proposed scheme achieves the diversity order of $R\min\{N_s, N_d\}$, where R, N_s, and N_d are the number of relays, source antennas, and the destination antennas, respectively.
- We verify the obtained analytical results using simulations. Results show that the derived error rates have the same system performance as simulation results. Assuming $R = 2$, $N_s = 2$, $N_d = 2$, the proposed opportunistic scheme outperforms DSTC by about 2 dB gain in SNR at BER of 10^{-5}.

Notations: The superscripts ^{t} and ^{H} stand for transposition

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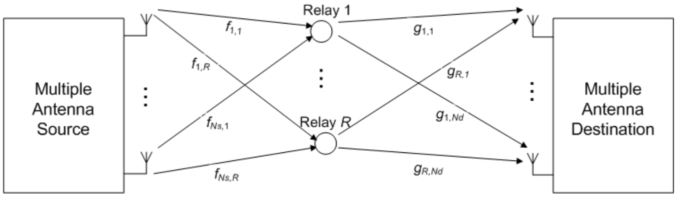

Fig. 1. Wireless relay network including one source with N_s antennas, R relays, and one destination with N_d antennas.

and conjugate transposition, respectively. The expectation operation is denoted by $\mathbb{E}\{\cdot\}$. The symbol $\boldsymbol{I}_T$ stands for the $T \times T$ identity matrix. $\|\boldsymbol{A}\|$ denotes the Frobenius norm of the matrix $\boldsymbol{A}$. The trace of the matrix $\boldsymbol{A}$ is denoted by $\text{tr}\,\{\boldsymbol{A}\}$. $\text{diag}\{\boldsymbol{A}_1, \ldots, \boldsymbol{A}_R\}$ denotes the block diagonal matrix.

II. System Model

Consider a wireless communication scenario where the source node s transmits information to the destination node d with the assistance of one or more relays denoted Relay $r = 1, 2, \ldots, R$ (see Fig. 1). The source and destination nodes are equipped with N_s and N_d antennas, respectively. Without loss of generality, it is assumed that each relay nodes is equipped with a single antennas. Note that this network can be transformed to relays with multiple antenna, since the transmit and receive signals at different antennas of the same relay can be processed and designed independently.

We denote the links from N_s antennas of the source to the rth relay as $f_{1,r}, f_{2,r}, \ldots, f_{N_s,r}$, and the links from the rth relay to the N_d antennas at the destination as $g_{r,1}, g_{r,2}, \ldots, g_{r,N_d}$. Under the assumption that each link undergoes independent Rayleigh process $f_{i,r}$, and $g_{r,j}$ are independent complex Gaussian random variables with zero-mean and variances $\sigma^2_{f_r}$, and $\sigma^2_{g_r}$, respectively. Since multiple antennas in source and destination are co-located, and the co-located antennas have approximately the same distances to relays, we skipped the i and j indices of $\sigma^2_{f_r}$ and $\sigma^2_{g_r}$.

Assume that the source wants to send K symbols s_1, s_2, ..., s_K to the destination during T time slots. T should be less than the coherent interval, that is, the time duration among which channels $f_{i,r}$, and $g_{r,j}$ keep constant. Henceforth, we assume using full-rate space-time codes, and thus, $K = T$. Similar to [1], our scheme requires two phases of transmission. During the first phase, the source should transmit a $T \times N_s$ dimensional orthogonal code matrix $\boldsymbol{S}_1$ to *all* relays. We can represent $\boldsymbol{S}_1$ in terms of the vector $\boldsymbol{s} = [s_1, s_2, \ldots, s_T]^t$, consisting of T information symbols as

$$\boldsymbol{S}_1 = [\boldsymbol{A}_1\boldsymbol{s}\;\boldsymbol{A}_2\boldsymbol{s}\;\ldots\;\boldsymbol{A}_{N_s}\boldsymbol{s}], \tag{1}$$

where $\boldsymbol{A}_i$, $i = 1, \ldots, N_s$, are $T \times T$ unitary matrices, and $\boldsymbol{s}_i = \boldsymbol{A}_i\boldsymbol{s}$ describes the ith column of a $T \times N_s$ orthogonal space-time code. We assume the following normalization

$$\mathbb{E}\left[\text{tr}\{\boldsymbol{S}_1^H\boldsymbol{S}_1\}\right] = \mathbb{E}\left[\text{tr}\left\{\sum_{k=1}^{T}|s_k|^2\boldsymbol{I}_{N_s}\right\}\right] = N_s. \tag{2}$$

The source transmits $\sqrt{P_1T/N_s}\boldsymbol{S}_1$ where P_1T is the average total power used at the source during the first phase. Thus, $\sqrt{P_1T/N_s}\boldsymbol{s}_i$, $i = 1, \ldots, N_s$, is the signal sent by the ith antenna with the average power of P_1T/N_s. Assuming that $f_{i,r}$ does not vary during T successive intervals, the $T \times 1$ receive signal vector at the rth relay is

$$\boldsymbol{x}_r = \sqrt{\frac{P_1T}{N_s}}\boldsymbol{S}_1\boldsymbol{f}_r + \boldsymbol{v}_r, \tag{3}$$

where $\boldsymbol{f}_r = [f_{1,r}\; f_{2,r}\; \ldots\; f_{N_s,r}]^t$, and $\boldsymbol{v}_r$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with variance $\mathcal{N}_1$.

In the second phase of the transmission, all relays simultaneously transmit linear functions of their received signals $\boldsymbol{x}_r$. In order to construct a distributed space-time codes, the received signal at the jth antenna of the *destination* is collected inside the $T \times 1$ vector $\boldsymbol{y}_j$ as

$$\boldsymbol{y}_j = \sum_{r=1}^{R} g_{r,j}\,\rho_r\boldsymbol{C}_r\boldsymbol{x}_r + \boldsymbol{w}_j, \tag{4}$$

for $j = 1, 2, \ldots, N_r$, where $\boldsymbol{w}_j$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with component-wise variance $\mathcal{N}_2$, ρ_r is the scaling factor at relay r, and $\boldsymbol{C}_r$, of size $T \times T$, are obtained by representing the rth column of an appropriate $T \times R$ dimensional space-time code matrix as $\boldsymbol{C}_r\boldsymbol{s}$. This construction method originates from the construction of a space-time code for co-located multiple-antenna systems, where the transmitted signal vector from the kth antenna is $\boldsymbol{C}_k\boldsymbol{s}$ [8]. When there is no instantaneous channel state information (CSI) at the relays, but statistical CSI is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is, $\rho_r = \sqrt{\frac{P_{2,r}}{\sigma^2_{f_r}P_1 + \mathcal{N}_1}}$ where $P_{2,r}$ is the average transmitted power from Relay r.

We can further represent input-output relationship of the DSTC as the space-time code in a multiple-antenna system. By setting the $T \times N_sR$ space-time encoded signal

$$\boldsymbol{S} = [\boldsymbol{C}_1\boldsymbol{S}_1,\, \boldsymbol{C}_2\boldsymbol{S}_1,\, \ldots,\, \boldsymbol{C}_R\boldsymbol{S}_1], \tag{5}$$

and by concatenating the received signals of the destination antennas, i.e., $\boldsymbol{Y} = [\boldsymbol{y}_1\,\boldsymbol{y}_2\,\ldots\,\boldsymbol{y}_{N_d}]$, from (3)-(4), we have

$$\boldsymbol{Y} = \sqrt{\frac{P_1T}{N_s}}\boldsymbol{S}\boldsymbol{H} + \boldsymbol{W}_T, \tag{6}$$

where the $N_sR \times N_d$ channel matrix $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \boldsymbol{F}\boldsymbol{\Lambda}\boldsymbol{G},$$

$$\boldsymbol{F} = \text{diag}\{\boldsymbol{f}_1, \ldots, \boldsymbol{f}_R\}, \qquad \boldsymbol{\Lambda} = \text{diag}\{\rho_1, \ldots, \rho_R\},$$

$$\boldsymbol{g}_r = [g_{r,1}\; g_{r,2}\; \ldots\; g_{r,N_d}], \qquad \boldsymbol{G} = \left[\boldsymbol{g}_1^t, \ldots, \boldsymbol{g}_R^t\right]^t,$$

and the noise is collected into the $T \times N_d$ matrix

$$\boldsymbol{W}_T = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{G} + \boldsymbol{W}. \tag{7}$$

where $\boldsymbol{V} = [\boldsymbol{C}_1\boldsymbol{v}_1\,\boldsymbol{C}_2\boldsymbol{v}_2\,\ldots\,\boldsymbol{C}_R\boldsymbol{v}_R]$ and $\boldsymbol{W} = [\boldsymbol{w}_1\,\boldsymbol{w}_2\,\ldots\,\boldsymbol{w}_{N_d}]$. Now, we derive the covariance of $\boldsymbol{W}_T$ which will be used for calculating the received SNR at the destination. Since $\boldsymbol{v}_r$ and $\boldsymbol{w}_j$ are independent complex Gaussian random variables, which are jointly independent, the conditional auto-covariance matrix of $\boldsymbol{W}_T$ can be shown to be

$$\begin{aligned}\mathrm{Cov}(\boldsymbol{W}_T|\boldsymbol{F},\boldsymbol{G}) &= \mathbb{E}_v[\boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{G}\boldsymbol{G}^H\boldsymbol{\Lambda}^H\boldsymbol{V}^H] + \mathbb{E}[\boldsymbol{W}\boldsymbol{W}^H] \\ &= \sum_{r=1}^{R}\rho_r^2\|\boldsymbol{g}_r\|^2\mathcal{N}_1\boldsymbol{C}_r\boldsymbol{C}_r^H + N_d\mathcal{N}_2\boldsymbol{I}_T \\ &= \left(\sum_{r=1}^{R}\rho_r^2\|\boldsymbol{g}_r\|^2\mathcal{N}_1 + N_d\mathcal{N}_2\right)\boldsymbol{I}_T. \end{aligned} \tag{8}$$

Thus, the noise vector $\boldsymbol{W}_T$ is white. The third equality in (8) follows from the fact that for each relay, $\boldsymbol{C}_r$ is a unitary matrix.

Since in this paper, we focus on orthogonal design, the maximum likelihood (ML) detection is decomposed to single-symbol detection, and by the fact that noise in (8) is white, maximal-ratio combining (MRC) can be applied at the destination. To calculate the post detection SNR at the output of the ML DSTC decoder, we need to compute the received signal power. Hence, using (6), we have

$$\eta_{s_d} = \frac{P_1T}{N_s}\mathbb{E}_s\Big[\mathrm{tr}\{\boldsymbol{S}\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{S}^H\}\Big] = \frac{P_1T}{N_s}\,\mathrm{tr}\{\boldsymbol{H}\boldsymbol{H}^H\,\mathbb{E}_s[\boldsymbol{S}^H\boldsymbol{S}]\}. \tag{9}$$

To have single-symbol ML detection, we should design distributed DSTC, such that

$$\boldsymbol{S}^H\boldsymbol{S} = (|s_1|^2 + |s_2|^2 + \ldots + |s_T|^2)\boldsymbol{I}_{N_sR}, \tag{10}$$

and using the normalization assumed in (2), we have

$$\mathbb{E}_s[\boldsymbol{S}^H\boldsymbol{S}] = \boldsymbol{I}_{N_sR}. \tag{11}$$

Thus, η_{s_d} in (9) can be evaluated as

$$\begin{aligned}\eta_{s_d} &= \frac{P_1T}{N_s}\,\mathrm{tr}\{\boldsymbol{H}\boldsymbol{H}^H\} = \frac{P_1T}{N_s}\sum_{i=1}^{N_sR}\Big[\boldsymbol{H}\boldsymbol{H}^H\Big]_{i,i} \\ &= \frac{P_1T}{N_s}\sum_{r=1}^{R}\sum_{n=1}^{N_s}|f_{n,r}|^2\rho_r^2\sum_{j=1}^{N_d}|g_{r,j}|^2 = \frac{P_1T}{N_s}\sum_{r=1}^{R}\rho_r^2\|\boldsymbol{f}_r\|^2\|\boldsymbol{g}_r\|^2. \end{aligned} \tag{12}$$

III. Power Control in A&F Space-Time Coded Cooperation

In this section, we propose power allocation schemes for the A&F distributed space-time codes with multiple antennas source/destination, based on maximizing the received SNR at the destination d.

The conditional variance of the equivalent received noise is obtained in (8). Thus, using (8) and (12), the instantaneous received SNR at the destination can be written as

$$\begin{aligned}\mathrm{SNR}_{\text{ins}} &= \frac{\sum_{k=1}^{R}P_1\|\boldsymbol{f}_k\|^2\|\boldsymbol{g}_k\|^2\frac{P_{2,k}}{\sigma_{f_k}^2P_1+\mathcal{N}_1}}{N_s\sum_{k=1}^{R}\|\boldsymbol{g}_k\|^2\frac{P_{2,k}}{\sigma_{f_k}^2P_1+\mathcal{N}_1}\mathcal{N}_1 + N_sN_d\mathcal{N}_2} \\ &= \frac{\boldsymbol{p}^t\boldsymbol{U}\boldsymbol{p}}{\boldsymbol{p}^t\boldsymbol{Q}\boldsymbol{p} + N_sN_d\mathcal{N}_2}, \end{aligned} \tag{13}$$

where $\boldsymbol{p} = [\sqrt{P_{2,1}}, \sqrt{P_{2,2}}, \ldots, \sqrt{P_{2,R}}]^t$ and diagonal matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are defined as

$$\begin{aligned}\boldsymbol{U} &= \mathrm{diag}\left[\frac{P_1\|\boldsymbol{f}_1\|^2\|\boldsymbol{g}_1\|^2}{\sigma_{f_1}^2P_1+\mathcal{N}_1}, \frac{P_1\|\boldsymbol{f}_2\|^2\|\boldsymbol{g}_2\|^2}{\sigma_{f_2}^2P_1+\mathcal{N}_1}, \ldots, \frac{P_1\|\boldsymbol{f}_R\|^2\|\boldsymbol{g}_R\|^2}{\sigma_{f_R}^2P_1+\mathcal{N}_1}\right], \\ \boldsymbol{Q} &= \mathrm{diag}\left[\frac{N_s\|\boldsymbol{g}_1\|^2\mathcal{N}_1}{\sigma_{f_1}^2P_1+\mathcal{N}_1}, \frac{N_s\|\boldsymbol{g}_2\|^2\mathcal{N}_1}{\sigma_{f_2}^2P_1+\mathcal{N}_1}, \ldots, \frac{N_s\|\boldsymbol{g}_R\|^2\mathcal{N}_1}{\sigma_{f_R}^2P_1+\mathcal{N}_1}\right]. \end{aligned} \tag{14}$$

Then, the optimization problem is formulated as

$$\boldsymbol{p}^* = \arg\max_{\boldsymbol{p}}\mathrm{SNR}_{\text{ins}}, \quad \text{subject to } \boldsymbol{p}^t\boldsymbol{p} = P_2. \tag{15}$$

where the $R \times 1$ vector $\boldsymbol{p}^*$ denotes the optimum values of power control coefficients. Moreover, since $\boldsymbol{p}^t\boldsymbol{p} = P_2$, we can rewrite (13) as $\mathrm{SNR}_{\text{ins}} = \frac{\boldsymbol{p}^t\boldsymbol{U}\boldsymbol{p}}{\boldsymbol{p}^t\boldsymbol{W}\boldsymbol{p}}$, where diagonal matrix $\boldsymbol{W}$ is defined as $\boldsymbol{W} = \boldsymbol{Q} + \frac{N_sN_d\mathcal{N}_2}{P_2}\boldsymbol{I}_T$. Since $\boldsymbol{W}$ is a positive semi-definite matrix, we define $\boldsymbol{q} \triangleq \boldsymbol{W}^{\frac{1}{2}}\boldsymbol{p}$, where $\boldsymbol{W} = (\boldsymbol{W}^{\frac{1}{2}})^t\boldsymbol{W}^{\frac{1}{2}}$. Then, $\mathrm{SNR}_{\text{ins}}$ can be rewritten as

$$\mathrm{SNR}_{\text{ins}} = \frac{\boldsymbol{q}^t\boldsymbol{Z}\boldsymbol{q}}{\boldsymbol{q}^t\boldsymbol{q}}, \tag{16}$$

where diagonal matrix $\boldsymbol{Z}$ is $\boldsymbol{Z} = \boldsymbol{U}\boldsymbol{W}^{-1}$. Now, using Rayleigh-Ritz theorem [9], we have $\frac{\boldsymbol{q}^t\boldsymbol{Z}\boldsymbol{q}}{\boldsymbol{q}^t\boldsymbol{q}} \leq \lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of $\boldsymbol{Z}$, which is corresponding to the largest diagonal element of $\boldsymbol{Z}$, i.e.,

$$\begin{aligned}\lambda_{\max} &= \max_{i\in\{1,\ldots,R\}}\frac{P_1P_2\|\boldsymbol{f}_i\|^2\|\boldsymbol{g}_i\|^2}{P_2N_s\|\boldsymbol{g}_i\|^2\mathcal{N}_1 + N_sN_d\mathcal{N}_2(\sigma_{f_i}^2P_1+\mathcal{N}_1)} \\ &\triangleq \max_{i\in\{1,\ldots,R\}}\gamma_i. \end{aligned} \tag{17}$$

The equality in $\frac{\boldsymbol{q}^t\boldsymbol{Z}\boldsymbol{q}}{\boldsymbol{q}^t\boldsymbol{q}} = \lambda_{\max}$ holds if $\boldsymbol{q}$ is proportional to the eigenvector of $\boldsymbol{Z}$ corresponding to $\lambda_{\max}$. Using the eigenvalue decomposition of the diagonal matrix $\boldsymbol{Z}$, it is obvious that the matrix which is consisting of the normalized eigenvectors, is the identity matrix. Hence, the optimum $\boldsymbol{q}_{\max}$ is proportional to $\boldsymbol{e}_{i_{\max}}$, which is a $R \times 1$ vector with only zero elements, except one at the $i_{\max}$-th component. On the other hand, since $\boldsymbol{p} = \boldsymbol{W}^{-\frac{1}{2}}\boldsymbol{q}$, and $\boldsymbol{W}$ is a diagonal matrix, the optimum $\boldsymbol{p}^*$ is also proportional to $\boldsymbol{e}_{i_{\max}}$. Using the power constraint of the transmitted power in the second phase, i.e., $\boldsymbol{p}^t\boldsymbol{p} = P_2$, we have $\boldsymbol{p}^* = \sqrt{P_2}\boldsymbol{e}_{i_{\max}}$. This means that for each realization of the network channels, the best relay should transmit all the available power P_2 and all other relays should stay silent.

The process of selecting the best relay could be done by the destination. This is feasible since the destination node should be aware of both backward and forward channels for coherent decoding. Thus, the same channel information could be exploited for the purpose of relay selection. However, if we assume a distributed relay selection algorithm, in which relays independently decide to select the best relay among them, such as in [7], the knowledge of local channels f_i and g_i is required in the ith relay. The estimation of f_i and g_i can be done by transmitting a ready-to-send (RTS) packet and a clear-to-send (CTS) packet in MAC protocols.

IV. SER Performance Analysis

In this section, we will derive the SER formulas of the best relay selection strategy under the amplify-and-forward mode. We should first derive the PDF of γ_r in (17).

Proposition *1:* For γ_r in (17), the probability density function $p_r(\gamma)$ can be written as

$$p_r(\gamma)=\sum_{k=0}^{N_s}\frac{2a^k\overline{Y}_r^k\binom{N_s}{k}e^{-\frac{a\gamma}{\overline{X}_r}}}{\gamma\,(N_d-1)!\,(N_s-1)!\,b_r^k}\left(\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}\right)^{\frac{\mu+k}{2}}K_{\nu+k}\left(2\sqrt{\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}}\right), \quad (18)$$

where $\mu = N_s + N_d$, $\nu = N_d - N_s$, $\overline{X}_r = N_s\sigma_{f_r}^2$, $\overline{Y}_r = N_d\sigma_{g_r}^2$, $a = \frac{N_s\mathcal{N}_1}{P_1}$, $b_r = \frac{N_sN_d\mathcal{N}_2(\sigma_{f_r}^2P_1+\mathcal{N}_1)}{P_1P_2}$, and $K_n(x)$ is the modified Bessel function of the second kind of order n.

Proof: Suppose $X = \|\boldsymbol{f}_r\|^2$ and $Y = \|\boldsymbol{g}_r\|^2$, then X and Y are gamma distributed [10, Eq. (5.14)] with mean of $\overline{X}_r$ and $\overline{Y}_r$, respectively. Therefore, similar to the approach used in [11, Eq. (3)], the PDF of $\gamma_r = XY/(aY + b_r)$ can be calculated as

$$\begin{aligned}p_r(\gamma) &= \frac{\overline{X}_r^{-N_s}\gamma^{N_s-1}e^{-\frac{a\gamma}{\overline{X}_r}}}{\overline{Y}_r^{N_d}(N_d-1)!\,(N_s-1)!}\\ &\quad\times\int_0^\infty y^{N_d-N_s-1}(ay+b_r)^{N_s}e^{-\left(\frac{b_r\gamma}{\overline{X}_ry}+\frac{y}{\overline{Y}_r}\right)}dy\\ &= \left(\frac{b_r}{\overline{X}_r}\right)^{N_s}\frac{\gamma^{N_s-1}e^{-\frac{a\gamma}{\overline{X}_r}}}{\overline{Y}_r^{N_d}(N_d-1)!\,(N_s-1)!}\\ &\quad\times\sum_{k=0}^{N_s}\binom{N_s}{k}\int_0^\infty y^{N_d-N_s-1}\left(\frac{ay}{b_r}\right)^k e^{-\left(\frac{b_r\gamma}{\overline{X}_ry}+\frac{y}{\overline{Y}_r}\right)}dy.\end{aligned} \quad (19)$$

where we have used binomial theorem [12, Eq. (2.36)] in the second equality. Thus, the PDF of γ_r can be found by solving the integral in (19) using [13, Eq. (3.471)], yielding (18). ■

Define $\gamma_{\max} \triangleq \max\{\gamma_1, \gamma_2, \ldots, \gamma_R\}$. The conditional SER of the best relay selection system under A&F mode with R relays can be written as $P_e(R|\boldsymbol{F},\boldsymbol{G}) = c\,Q\left(\sqrt{g\,\gamma_{\max}}\right)$, where parameters c and g are represented as

$$c_{\text{QAM}}=4\frac{\sqrt{M}-1}{\sqrt{M}}, c_{\text{PSK}}=2, g_{\text{QAM}}=\frac{3}{M-1}, g_{\text{PSK}}=2\sin^2\!\left(\frac{\pi}{M}\right).$$

Using the result from order statistics, and by assuming that all channel coefficients are independent of each other, the PDF of $\gamma_{\max}$ can be written as

$$p_{\max}(\gamma) = \sum_{r=1}^{R}p_r(\gamma)\prod_{\substack{j=1\\ j\neq r}}^{R}\Pr\{\gamma_j < \gamma\}. \quad (20)$$

where $\Pr\{\gamma_j < \gamma\}$ can be evaluated as

$$\begin{aligned}&\Pr\{\gamma_j < \gamma\} = \Pr\{XY/(aY + b_j) < \gamma\}\\ &= \int_0^\infty\left(1-e^{-\frac{x}{\overline{X}_j}}\sum_{n=0}^{N_s-1}\frac{1}{n!}\left(\frac{x}{\overline{X}_j}\right)^n\right)\frac{y^{N_d-1}}{(N_d-1)!\,\overline{Y}_j^{N_d}}e^{-\frac{y}{\overline{Y}_j}}dy\\ &= 1-\sum_{n=0}^{N_s-1}\sum_{k=0}^{n}\frac{\binom{n}{k}a^kb_j^{n-k}\gamma^ne^{-\frac{a\gamma}{\overline{X}_j}}}{n!\,(N_d-1)!\,\overline{X}_j^n\overline{Y}_j^{N_d}}\int_0^\infty y^{N_d+k-n-1}e^{-\frac{y}{\overline{Y}_j}-\frac{b_j\gamma}{y\overline{X}_j}}dy\\ &= 1-\sum_{n=0}^{N_s-1}\sum_{k=0}^{n}\frac{2\binom{n}{k}a^k\overline{Y}_j^ke^{-\frac{a\gamma}{\overline{X}_j}}}{n!\,(N_d-1)!\,b_j^k}\left(\frac{b_j\gamma}{\overline{X}_j\overline{Y}_j}\right)^{\frac{N_d+n+k}{2}}K_{N_d-n+k}\left(2\sqrt{\frac{b_j\gamma}{\overline{X}_j\overline{Y}_j}}\right),\end{aligned} \quad (21)$$

where $x = \frac{\gamma(ay+b_j)}{y\overline{X}_j}$, and in the second equality, we used Erlang distribution [12, (3.48)].

The SER expression for the selection relaying scheme discussed in Section III is now derived. Averaging over conditional SER $P_e(R|\boldsymbol{F},\boldsymbol{G})$, we have the exact SER expression as

$$\begin{aligned}P_e(R) &= \int_0^\infty P_e(R|\boldsymbol{F},\boldsymbol{G})\,p_{\max}(\gamma)\,d\gamma\\ &= \int_0^\infty c\,Q\left(\sqrt{g\,\gamma}\right)\,p_{\max}(\gamma)\,d\gamma.\end{aligned} \quad (22)$$

Next, a closed-form SER formula for the case of $N_d \neq N_s$ is derived, which is valid in the high SNR regime. This simple expression can be used for a power allocation strategy among the cooperative nodes, or to get insight into the diversity-multiplexing tradeoff of the system.

Using the facts that $K_0(x) \approx -\ln(x)$ [14, Eq. (9.6.8)], $K_\nu(x) \approx \frac{1}{2}\Gamma(\nu)\left(\frac{x}{2}\right)^{-\nu}$, $\nu \neq 0$ [14, Eq. (9.6.9)], as $x \to 0$, where $\Gamma(\nu)$ is gamma function of order ν, and $K_\nu(x) = K_{-\nu}(x)$ [14, Eq. (9.6.6)], the $p_r(\gamma)$ in (18) can be approximated as

$$p_r(\gamma) \approx \sum_{k=0}^{N_s}\frac{\binom{N_s}{k}a^k\overline{Y}_r^k\Gamma(N_d-N_s+k)}{(N_s-1)!\,(N_d-1)!\,b_r^k}\frac{e^{-\frac{a\gamma}{\overline{X}_r}}}{\gamma}\left(\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}\right)^{N_s}, \quad (23)$$

for $N_d > N_s$, and

$$\begin{aligned}p_r(\gamma)\approx &\sum_{k=0}^{N_s-N_d-1}\frac{\binom{N_s}{k}a^k\overline{Y}_r^k\Gamma(N_s-N_d-k)}{(N_s-1)!\,(N_d-1)!\,b_r^k}\frac{e^{-\frac{a\gamma}{\overline{X}_r}}}{\gamma}\left(\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}\right)^{k+N_d}\\ &-\frac{2\binom{N_s}{N_s-N_d}(a\overline{Y}_r)^{N_s-N_d}}{(N_s-1)!\,(N_d-1)!\,b_r^{N_s-N_d}}\frac{e^{-\frac{a\gamma}{\overline{X}_r}}}{\gamma}\left(\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}\right)^{N_s}\ln\left(2\sqrt{\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}}\right)\\ &+\sum_{k=N_s-N_d+1}^{N_s}\frac{\binom{N_s}{k}a^k\overline{Y}_r^k\Gamma(N_d-N_s+k)}{(N_s-1)!\,(N_d-1)!\,b_r^k}\frac{e^{-\frac{a\gamma}{\overline{X}_r}}}{\gamma}\left(\frac{b_r\gamma}{\overline{X}_r\overline{Y}_r}\right)^{N_s},\end{aligned} \quad (24)$$

for $N_d < N_s$.

Lemma *1:* Let $N \triangleq \min\{N_s, N_d\}$ and $N_s \neq N_d$. The nth $(n < N-1)$ order derivatives of $p_r(\gamma)$ with respect to γ at zero are null, while the $(N-1)$th order derivative of $p_r(\gamma)$ with respect to γ) at zero is computed as

$$\begin{aligned}\frac{\partial^{N-1}p_r}{\partial\gamma^{N-1}}(0) &= \begin{cases}\sum_{k=0}^{N_s}\frac{\binom{N_s}{k}a^kb_r^{N_s-k}\,\Gamma(N_d-N_s+k)}{(N_d-1)!\,\overline{X}_r^{N_s}\overline{Y}_r^{N_s-k}}, & \text{if } N=N_s,\\ \frac{\Gamma(N_s-N_d)}{(N_s-1)!}\left(\frac{b_r}{\overline{X}_r\overline{Y}_r}\right)^{N_d}, & \text{if } N=N_d,\end{cases}\\ &\triangleq \Phi_{N_s,N_d,r}\end{aligned} \quad (25)$$

Proof: By applying the chain rule for differentiating composite functions into $p_r(\gamma)$ in (23)-(24), the desired result in (25) is obtained. ■

Lemma *2:* All the derivatives of the PDF of $\gamma_{\max}$, i.e., p_γ, evaluated at zero up to order $(NR-1)$ are zero, while the NR-th order derivative is given by

$$\frac{\partial^{NR}p_{\max}}{\partial\gamma^{NR}}(0) = R\prod_{r=1}^{R}\frac{\partial^{N-1}p_r}{\partial\gamma^{N-1}}(0). \quad (26)$$

Proof: Since γ_r has positive values, it is obvious that $\Pr\{\gamma_r < 0\} = 0$. Therefore, using (20) and Lemma 1, and by applying the chain rule when differentiating composite functions, it can be shown that the derivatives of the PDF of $p_{\max}$, evaluated at zero up to order $(NR-1)$ are zero.

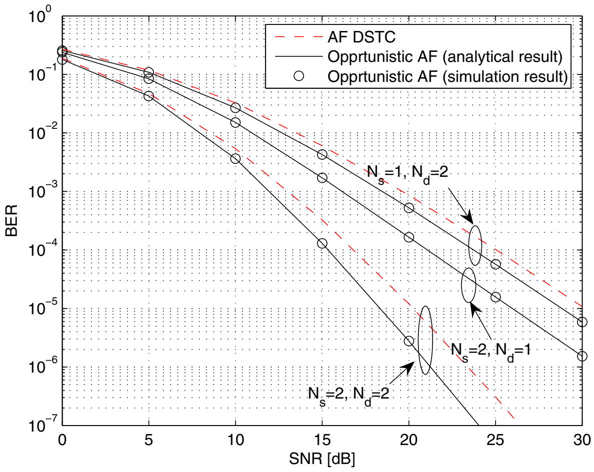

Fig. 2. Performance comparison of analytical and simulated results of a relay network with BPSK signals, $R = 2$, and $T = 4$.

In addition, $\frac{\partial^{NR} p_{\max}}{\partial \gamma^{NR}}(0)$ has a limited nonzero value when $N_s \neq N_d$ given by (25), which completes the proof. ■

Asymptotic expression for the SER of the system is presented in the following proposition.

Proposition *2:* Suppose the relay network consisting of R relays with the multiple antennas source and destination. The SER of this system at high SNRs can be calculated as

$$P_e(R) \approx \frac{\prod_{i=1}^{NR+1}(2i-1)}{2g^{NR+1}} \frac{cR}{(NR+1)!} \prod_{r=1}^{R} \Phi_{N_s,N_d,r}. \quad (27)$$

Proof: To deduce the asymptotic behavior of the average SER, we are using the approximate expression given in [8]. When the derivatives of $p_\gamma(\gamma)$ up to ν-th order are null at $\gamma = 0$, then the SER at high SNRs can be given by

$$P_e(R) \approx \frac{\prod_{i=1}^{NR+1}(2i-1)}{2(NR+1)g^{NR+1}} \frac{c}{(NR)!} \frac{\partial^{NR} p_{\max}}{\partial \gamma^{NR}}(0). \quad (28)$$

Applying Lemmas 2, we have

$$P_e(R) \approx \frac{\prod_{i=1}^{NR+1}(2i-1)}{2(NR+1)g^{NR+1}} \frac{cR}{(NR)!} \prod_{r=1}^{R} \frac{\partial^{N-1} p_r}{\partial \gamma^{N-1}}(0). \quad (29)$$

Combining (25) and (29), (27) is obtained. ■

Corollary *1:* The A&F opportunistic relaying scheme with multiple antennas source and destination over Rayleigh fading provides the diversity gain of $R\min\{N_s, N_d\}$.

Proof: A tractable definition of the diversity gain is $G_d = -\lim_{\mu\to\infty} \frac{\log(P_e(R))}{\log(\mu)}$, [15, Eq. (1.19] where μ denotes the transmit SNR. Now, using (25) and (27), it is easy to show that the diversity order G_d becomes $R\min\{N_s, N_d\}$. ■

V. Simulation Results

In this section, the performance of the distributed space-time codes are compared with opportunistic relaying in A&F mode through simulations. The signal symbols are modulated as BPSK. We fixed the total power consumed in the whole network as P and use the equal power allocation, i.e, $P_1 = P_2 = \frac{P}{2}$. Assume the relays and the destination have the same value of noise power, i.e., $\mathcal{N}_1 = \mathcal{N}_2$, and all the links have unit-variance Rayleigh flat fading. We assume $T = 4$, and we use the orthogonal space-time code structure in (1) and (5). For the case of $N_s = 2$, $R = 2$, the matrices used at the relays are $\boldsymbol{A}_1 = \boldsymbol{C}_1 = \boldsymbol{I}_4$, and

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \boldsymbol{C}_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \quad (30)$$

In Fig. 2, the BER performance of the A&F DSTC is compared to the proposed A&F opportunistic relaying, when $R = 2$. For A&F DSTC, equal power allocation is used among the relays. One can observe from Fig. 2 that the A&F opportunistic scheme achieve around 2 dB gain in SNR at BER 10^{-5}. Furthermore, Fig. 2 confirms that the analytical results attained in (22) for finding SER for A&F opportunistic relaying with space-time coded source node have the same performance as the simulation results. Observing the curves behavior in high SNR, it can be seen that the diversity order of the system becomes $R\min\{N_s, N_d\}$.

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