Device-to-device (D2D) communication is considered as a promising resource reuse in cellular networks. However, the allocation of channel resources and power to D2D communication requires precise coordination since D2D user equipments (UEs) can cause interference to cellular UEs. In this paper, we develop Nash Bargaining Solution (NBS) and Nash Competitive games frameworks to model the power allocation problem. NBS is a type of cooperative games, which has been applied for solving resource allocation problems among competing players. In the NBS game, the optimization problem is not convex. We propose a method to make the problem convex. In the Nash Competitive game, the problem is convex so that we apply KKT conditions and find a closed form solution for Nash Equilibrium (NE). Finally, we perform computer simulations to study the performance of the games. Simulation results show that our method for finding NBS has a significant accuracy. Results also indicate that D2D users have higher rates in NBS game compared to Nash Competitive game which states that the cooperation helps D2D users to gain more efficiency.

Index Terms—Device-to-device (D2D) communication, power allocation, Nash Bargaining Solution (NBS), Nash Equilibrium (NE).

I. INTRODUCTION

Device-to-device (D2D) underlay communication has been proposed for cellular networks to improve spectrum efficiency and system sum rate [1]. In a D2D link, user equipments (UEs) can communicate with each other through a direct connection instead of using the base station (BS). D2D pairs (users) share the same bandwidth resources with the traditional UEs. Although D2D communication causes improvements in system capacity and spectral efficiency, it also brings undesirable interference to the cellular network as a result of spectrum sharing. Therefore, it is necessary to formulate an effective interference coordination to guarantee a target performance level of the cellular communication.

Some researchers performed studies on resource management to utilize D2D technology efficiently [2]-[3]. The proper power control method is stated in [2] in order to coordinate the interference to achieve sum rate increase. In [3], some cross-layer optimization and resource management techniques have been proposed. The pairing of the cellular and D2D users for sharing the same resources has been studied in [4] and [5]. This type of pairing improves the gain from intra-cell spectrum reuse. In [6], authors studied power control approaches by maximizing sum rate over signal to interference and noise ratio (SINR) of the cellular system. Feng et al. proposed a three-step algorithm to preserve QoS for both D2D and cellular UEs in [7]. Thus, the total throughput of the network has been improved. In [8], sum rate of D2D UEs are maximized subject to the point that each cellular UE has minimum required rate.

Recently, a large number of various games have been applied to study resource allocation problems. In [9] and [10], a reverse iterative combinatorial auction game is proposed to maximize the system sum rate over the resource reuse of multiple D2D pairs. In the auction, packages of D2D pairs and the corresponding transmit powers are goods and cellular channels are viewed as bidders competing to obtain rate increase. In [11], Stackelberg game has been used to model interactions between D2D and cellular UEs. It is a leader-follower-based game and its solution is achievable via the backward induction. Since this game is one of the competitive games, its outcome does not satisfy Pareto efficiency and is not socially optimum [12]. Thus, the results could be improved by applying cooperative games such as Nash Bargaining Solution (NBS). NBS is a type of cooperative games which has been used to solve resource allocation problem among competing players. In [13], the authors model NBS for studying interactions between one cellular UE and one D2D UE which is not reasonable and realistic assumption in practical environments. They also consider high SINR regime to solve the problem. The high SINR assumption indicates the case in which UEs have limited choices of coding schemes and modulations, and it is impossible to decode a transmission scheme if the SNR is below some threshold.

In this paper, we propose effective power allocation methods based on NBS and Nash Competitive games for D2D underlay cellular communication. In the NBS, D2D links cooperate with each other. However, in the Nash Competitive game, they are selfish and compete in order to maximize their own rates. One of the advantages of the NBS is that each D2D user receives at least a minimum amount of resources by cooperation and this motivates each user to cooperate instead of competing. In practical situations, D2D and cellular UEs can have cooperation and also competition in order to find their proper transmit powers. Thus, our work is practical in cellular communication networks. Formulating the games leads to two optimization problems which are NBS and Nash competitive problems.

The NBS optimization problem is not convex. Therefore, we first make the problem convex by our new method and then
find the solution. In contrast to [13], our approach works well for all ranges of SINR and also multiple D2D pairs. Unlike the NBS problem, the Nash Competitive optimization problem is convex. Thus, we apply KKT conditions and derive the closed form solution for Nash Equilibrium (NE). Simulation results show that the power and rate values for D2D UEs in NBS game are 99 percent similar to each other for non-convex and convexified problems. This proves that our method for making the problem convex works well. Moreover, simulation results state that D2D UEs have higher rates in NBS game compared to Nash Competitive game. This means it is better for D2D pairs to cooperate with each other instead of being selfish.

II. SYSTEM MODEL

In this paper, we consider a scenario of one cell with one BS and multiple users. As shown in Fig.1, we have two kinds of users, i.e., D2D users that have direct data transmissions and traditional cellular users that work in the uplink mode. Each user is equipped with a single omnidirectional antenna and is distributed uniformly in the cell. The two users in a D2D pair satisfy the distance constraint of D2D communication and different D2D pairs can have communicating demands at the same time.

Fig.1 illustrates a scenario in which cellular users work in uplink and share their resources with D2D pairs. $UE_{d1}$ and $UE_{d2}$ form a D2D pair while $UE_c$ is a cellular user. We assume $UE_{d1}$ is the D2D pair transmitter sharing the same spectrum resources with $UE_c$, and $UE_{d2}$ is the D2D pair receiver which receives interference from $UE_c$. The BS is exposed to interference from $UE_{d1}$. By letting more than one D2D pair reuse the same sub-channels, the performance of the system is improved. Therefore, we suppose that any resource blocks occupied by cellular users can be reassigned to multiple D2D pairs. Thus, D2D pairs also impose interference to each other. For the entire system, we assume the number of D2D pairs which decide to have communication is $D$ and the total number of cellular orthogonal sub-channels is $N$.

We model the channel as the Rayleigh fading channel. Thus, the channel response follows the independent identical complex Gaussian distribution. In addition, we used the free space propagation path-loss model, $P = P_0 \left( \frac{d}{d_0} \right)^{-\alpha}$, where $P_0$ and $P$ denote signal power measured at $d_0$ and $d$ away from the transmitter, respectively. The pathloss exponent is denoted by $\alpha$. Hence, the received power of each receiver can be expressed as

$$P_{r,ik} = P_i h_{ik} = P_i (d_{ik})^{-\alpha} h_{0}^2,$$

(1)

where $P_{r,ik}$ and $d_{ik}$ are the received power and the distance of the $i$-$k$ link, respectively. In addition, $P_i$ is the transmit power of equipment $i$ and $h_0$ is the complex Gaussian channel coefficient that follows the distribution $CN (0, 1)$. We assume the received power at $d_0$ is equal to transmit power.

Let $p_{ij}$ be the power allocated to D2D user (pair) $i$ at sub-channel $j$. The powers of D2D users should not be negative, i.e., $p_{ij} \geq 0$. The total power of user (pair) $i$ and other users (pairs) except $i$ over all sub-channels are denoted by $p_i$ and $p_{-i}$, respectively. The channel coefficients between D2D user $i$ and D2D user $k$ on the $j$th sub-channel is defined by $g_{ij}$. Besides, the channel coefficients between D2D user $i$ and BS is denoted by $h_{i,b}$. The additive white zero-mean Gaussian noise at D2D pair receiver $i$ on sub-channel $j$ is stated by $\sigma^2_{ij}$. Furthermore, $P_{int}$ denotes the total amount of interference from cellular users on D2D users in each sub-channel.

We suppose that the sum powers of each D2D user over all sub-channels is less than or equal to a threshold, which indicates $\sum_{j=1}^{N} p_{ij} \leq P_{i,max}, \forall i$. This is a reasonable assumption since each device has a limited transmit power which is usually around 23 dBm [14]. The D2D communication should not decrease the rates of cellular users significantly. Therefore, the rates of cellular users should be higher than a threshold. In order to satisfy this, the total interference occurred by D2D pairs on the BS must be limited, which states $\sum_{j=1}^{D} p_{ij} h_{i,b} \leq I_{th}^{j}, \forall j$, where $I_{th}^{j}$ is the maximum tolerable interference threshold on the $j$-th sub-channel for the BS. The rate of each D2D user is also as follows:

$$U_i (p_i, p_{-i}) = \sum_{j=1}^{N} \log_2 \left( 1 + \frac{g_{ij} p_{ij}}{\sigma^2_{ij} + \sum_{k \neq j} g_{ik} p_{kj} + P_{int}} \right).$$

(2)

III. NBS AND NASH COMPETITIVE GAMES FORMULATIONS AND OUR PROPOSED SOLUTIONS

In this section, we introduce NBS and Nash Competitive games for D2D underlay cellular network. Our goal is to find NBS and NE for these games, respectively. We suppose that powers of cellular users and their sub-channels are assigned and fixed. Hence, the total interference term from cellular users to a D2D user is constant and our optimization variables are transmit powers of D2D users. The NBS is a type of
cooperative games so that the D2D users cooperate with each other to maximize an objective function which is the product of their utility functions. In contrast, in Nash Competitive game, D2D users are selfish and intend to maximize their own rates.

A. Nash Bargaining Game and NBS

In this subsection, we introduce NBS for the D2D underlay cellular network. NBS satisfies Pareto efficiency and is socially optimum. Our goal is to find the NBS for our model. We consider each D2D user rate as its utility function. According to Nash’s axioms [12], the objective function would be the product of utility functions:

\[ U = \prod_{i=1}^{D} \left( U_i(p_i, p_{-i}) - U_i^{\text{min}} \right), \]  

(3)

where \( U_i^{\text{min}} \) is dependent on application and can have different values. In this paper, we assume that \( U_i^{\text{min}} \) is equal to zero for having fairness [15]. We also discussed the following constraints:

\[ \sum_{j=1}^{N} p_{ij} \leq P_{i,\text{max}} \forall i, \quad p_{ij} \geq 0 \forall i, j, \quad \sum_{i=1}^{D} p_{ij} h_{i,b} \leq I_j^{th} \forall j. \]  

(4)

Thus, the problem formulation can be written as:

\[(p_1^*, p_2^*, \ldots, p_D^*) = \max_{p} \prod_{i=1}^{D} U_i(p_i, p_{-i}) \]

subject to \( C1 : \sum_{j=1}^{N} p_{ij} \leq P_{i,\text{max}}, \forall i \)

\( C2 : p_{ij} \geq 0, \forall i, j \)

\( C3 : \sum_{i=1}^{D} p_{ij} h_{i,b} \leq I_j^{th} \forall j. \)  

(5)

Note that since NBS game is a type of cooperative games, we cannot treat the D2D interference on each other as noise. Therefore, the optimization problem in (5) has an objective function which is not concave with respect to \( p_{ij} \). Hence, although the constraints are linear and also convex, the optimization problem is not convex. Our goal is converting the problem to a convex problem so that we can derive a tractable solution. To this end, we use the following inequality for the rates of D2D users [16]:

\[ \alpha \log_2 x + \beta \leq \log_2 (1 + x). \]  

(6)

If we choose \( \alpha \) and \( \beta \) as following equations, then at point \( x_0 \) the inequality is changed to equality.

\[ \alpha = \frac{x_0}{1 + x_0}, \quad \beta = \log_2 (1 + x_0) - \frac{x_0}{1 + x_0} \log_2 x_0. \]  

(7)

In Fig. 2, we plot the \( \alpha \log_2 x + \beta \) and \( \log_2 (1 + x) \) functions versus \( x \) together in a same diagram for various values of \( x_0 \). According to the figure, for different values of \( x_0 \), these two functions are equal to each other at point \( x_0 \). If we set \( \alpha = 1 \) and \( \beta = 0 \), indeed we suppose that SINR \( \gg 1 \). In some papers such as [13], they assume that the SINR is very high. Thus, our method is more general and includes low SINR scenarios too. By applying (6) to the rate of each D2D user, we have:

\[ U_i(p_i, p_{-i}) = \prod_{j=1}^{N} \alpha_{ij} \log_2 \left( \frac{g_{ij} p_{ij}}{\sigma_i^2 + \sum_{k \neq i} g_{ij}^p p_{kj} + P_{\text{Int}}^i} \right) + \beta_{ij}. \]  

(8)

Note that the function is not concave with respect to \( p_{ij} \) yet. Now, we change the variable \( p_{ij} \) with \( e^{p_{ij}} \), and thus, the optimization problem is:

\[(\tilde{p}_1^*, \tilde{p}_2^*, \ldots, \tilde{p}_D^*) = \max_{\tilde{p}} \prod_{i=1}^{D} \left( \prod_{j=1}^{N} \alpha_{ij} \log_2 \left( \frac{g_{ij} e^{\tilde{p}_{ij}}}{\sigma_i^2 + \sum_{k \neq i} g_{ij}^p e^{\tilde{p}_{kj}} + P_{\text{Int}}^i} \right) + \beta_{ij} \right) \]

subject to \( C1 : \sum_{j=1}^{N} e^{\tilde{p}_{ij}} \leq P_{i,\text{max}}, \forall i \)

\( C2 : e^{\tilde{p}_{ij}} \geq 0, \forall i, j \)

\( C3 : \sum_{i=1}^{D} e^{\tilde{p}_{ij}} h_{i,b} \leq I_j^{th}, \forall j. \)  

(9)

Theorem 1: The optimization problem in (9) is a convex optimization problem.

Proof: The proof is provided in Appendix A.

Until now, we converted the non-convex problem to a convex problem which is the most challenging part in optimization problems. Now, we can apply KKT conditions to solve the convex problem. We update the values of \( \alpha \) and \( \beta \) in our convexified problem based on (7) right after the calculation of SINR in each iteration. This process makes our method to be very exact since after some iterations, the inequality in (6) is satisfied by an equality. We continue this updating process until the convergence.

B. Nash Competitive Game and NE

In this subsection, we introduce the Nash Competitive game for D2D underlay cellular network. Our goal is to derive the
The radius of the cell is 500m.

Isolated Cell, 1-sector

\[ A_{ij} = \sigma_i^2 + \sum_{k \neq i} g_{ik} p_{kj} + P_{\text{inc}}. \]  

(10)

where \( A_{ij} \) is the total interference of D2D and cellular users plus noise imposed to user \( i \) on sub-channel \( j \). Similar to the NBS game, we have the constraints stated in (4) for the Nash Competitive game. Thus, the problem formulation is modeled as:

\[
(p_1^*, p_2^*, ..., p_D^*) = \max_{p} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{g_{ij} p_{ij}}{A_{ij}} \right)
\]

subject to

\[ C1 : \sum_{j=1}^{N} p_{ij} \leq P_{i,\text{max}}, \quad \forall i \]

\[ C2 : p_{ij} \geq 0, \quad \forall i, j \]

\[ C3 : \sum_{i=1}^{D} p_{ij} h_{i,b} \leq I_j^b, \quad \forall j. \]  

(11)

The objective function in (11) is concave with respect to \( p_{ij} \). The constraints are linear and also convex. Thus, the optimization problem in (11) is convex. We write the Lagrangian function and KKT conditions for this convex problem as

\[ \lambda_i^* \geq 0, \quad \eta_j^* \geq 0, \]  

(12)

\[ \lambda_i \left( \sum_{j=1}^{N} p_{ij} - P_{i,\text{max}} \right) = 0, \quad \eta_j \left( \sum_{j=1}^{D} p_{ij} h_{i,b} - I_j^b \right) = 0, \]  

(13)

\[ \mathcal{L}(p_1^*, p_{-i}, \lambda_1^*, \eta_1^*) = \sum_{j=1}^{N} \log_2 \left( 1 + \frac{g_{ij} p_{ij}}{A_{ij}} \right) - \lambda_i \left( \sum_{j=1}^{N} p_{ij} - P_{i,\text{max}} \right) - \eta_j \left( \sum_{j=1}^{D} p_{ij} h_{i,b} - I_j^b \right), \]  

(14)

\[ \frac{\partial \mathcal{L}}{\partial p_{ij}} = 0 \Rightarrow p_{ij} = \left[ \frac{1}{\lambda_i^* + \eta_j^* h_{i,b}} \right] \ln 2 - \frac{A_{ij}}{g_{ij}^*} \]  

(15)

where \( \lambda_i^* \) and \( \eta_j^* \) are Lagrangian coefficients for C1 and C3 constraints, respectively. We update Lagrangian coefficients in every iteration and then calculate the D2D powers until their convergence iteratively. Both the subgradient and ellipsoid methods can be used in the update of dual variables [17]. Here, we choose the subgradient approach to update dual variables as following equations:

\[ \lambda_i^{(t+1)} = \lambda_i^{(t)} - \gamma_1^{(t)} (P_{i,\text{max}} - \sum_{j=1}^{N} p_{ij}) \]  

(16)

\[ \eta_j^{(t+1)} = \eta_j^{(t)} - \gamma_2^{(t)} \left( I_j^b - \sum_{j=1}^{D} p_{ij} h_{i,b} \right) \]  

(17)

where \( \gamma_1^{(t)} \) and \( \gamma_2^{(t)} \) are the step sizes of iteration \( t \), and they should satisfy the following condition,

\[ \sum_{t=1}^{\infty} \gamma_k^{(t)} = \infty, \quad \lim_{t \to \infty} \gamma_k^{(t)} = 0, \quad \forall k \in \{1, 2\}. \]  

(18)

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, simulations are carried out to evaluate the performance of the proposed solutions. We consider a single cell scenario as shown in Fig. 1. The wireless channel is modeled as Rayleigh fading channel. The main simulation parameters that follow typical values are summarized in Table I. We also consider \( I_j^b = 7.5 \times 10^{-14} \) (W) similar to [15] in our simulations.

For NBS game, we solve both non-convex and our convexified problems and compare the results. Powers of D2D pairs in different sub-channels for the convexified problem is stated in Table II. For non-convex problem, the solution is the local maximum of the problem which may not be the global maximum. Thus, we run it for a sufficient number of iterations and then choose the greatest value. Table III shows the D2D pairs powers in different sub-channels for the non-convex problem. We observe that the values in Table II and III are 99 percent similar to each other which proves that our convexified approach for finding NBS has a quite acceptable performance. For instance, we make bold power values of sub-channel 2 in Table II and III to confirm our good performance. D2D rates and sum rate values for convexified and non-convex problems are shown in Table IV. Their values are also 99 percent similar to each other as we expected.

For Nash Competitive game, we found the closed form for NE in (15). Fig. 3 and Fig. 4 show the powers and rates of D2D pairs versus the number of iterations, respectively. According to these figures, the powers of D2D pairs and their corresponding rates converge to specific values which proves that our approach reaches to the NE. By comparing Table IV values with Fig. 4, we find out that D2D users have higher rates in NBS game than in Nash Competitive game, which indicates that cooperating between D2D users provide them more efficiency.

Fig. 5 shows diagrams of the sum rate of D2D pairs versus the \( I_j^b \) for various values of \( P_{\text{max}} \) for NBS and Nash Competitive games. According to this figure, by increasing the \( P_{\text{max}} \), the sum rate of D2D pairs will increase which is reasonable. Based on this figure, the sum rate of D2D pairs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cellular Layout</td>
<td>Isolated Cell, 1-sector</td>
</tr>
<tr>
<td>System area</td>
<td>The radius of the cell is 500m</td>
</tr>
<tr>
<td>The maximum distance of D2D</td>
<td>50 m; 10-50 m</td>
</tr>
<tr>
<td>Noise spectral density</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>Sub-channel bandwidth</td>
<td>15 kHz</td>
</tr>
<tr>
<td>The maximum transmit power</td>
<td>BS: 46 dBm; Device: 23 dBm</td>
</tr>
<tr>
<td>Number of D2D pairs (D)</td>
<td>4</td>
</tr>
<tr>
<td>Number of Cellular UEs (C)</td>
<td>8</td>
</tr>
<tr>
<td>Number of sub-channels (N)</td>
<td>8</td>
</tr>
</tbody>
</table>
TABLE II. Powers of D2D pairs (dBm) in different sub-channels for convexified problem (NBS game).

<table>
<thead>
<tr>
<th>Sub-channel</th>
<th>D2D 1</th>
<th>D2D 2</th>
<th>D2D 3</th>
<th>D2D 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-channel 1</td>
<td>12.9986</td>
<td>14.3830</td>
<td>13.4060</td>
<td>14.4205</td>
</tr>
<tr>
<td>Sub-channel 2</td>
<td>14.3597</td>
<td>11.5715</td>
<td>15.8172</td>
<td>14.9959</td>
</tr>
<tr>
<td>Sub-channel 3</td>
<td>16.0697</td>
<td>15.3739</td>
<td>12.4546</td>
<td>13.4205</td>
</tr>
<tr>
<td>Sub-channel 4</td>
<td>14.6656</td>
<td>15.4243</td>
<td>10.1828</td>
<td>13.8387</td>
</tr>
<tr>
<td>Sub-channel 5</td>
<td>12.1131</td>
<td>9.7923</td>
<td>14.3370</td>
<td>15.6656</td>
</tr>
<tr>
<td>Sub-channel 6</td>
<td>12.5331</td>
<td>14.1165</td>
<td>14.5923</td>
<td>16.5408</td>
</tr>
<tr>
<td>Sub-channel 7</td>
<td>15.1105</td>
<td>13.6386</td>
<td>13.1865</td>
<td>0</td>
</tr>
<tr>
<td>Sub-channel 8</td>
<td>12.0235</td>
<td>14.7640</td>
<td>15.2337</td>
<td>12.5079</td>
</tr>
</tbody>
</table>

TABLE III. Powers of D2D pairs (dBm) in different sub-channels for non-convex problem (NBS game).

<table>
<thead>
<tr>
<th>Sub-channel</th>
<th>D2D 1</th>
<th>D2D 2</th>
<th>D2D 3</th>
<th>D2D 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-channel 1</td>
<td>12.9983</td>
<td>14.3830</td>
<td>13.4058</td>
<td>14.4205</td>
</tr>
<tr>
<td>Sub-channel 2</td>
<td>14.3596</td>
<td>11.5715</td>
<td>15.8171</td>
<td>14.9959</td>
</tr>
<tr>
<td>Sub-channel 3</td>
<td>16.0697</td>
<td>15.3738</td>
<td>12.4546</td>
<td>13.4205</td>
</tr>
<tr>
<td>Sub-channel 4</td>
<td>14.6654</td>
<td>15.4243</td>
<td>10.1828</td>
<td>13.8387</td>
</tr>
<tr>
<td>Sub-channel 5</td>
<td>12.1128</td>
<td>9.7923</td>
<td>14.3370</td>
<td>15.6657</td>
</tr>
<tr>
<td>Sub-channel 6</td>
<td>12.5329</td>
<td>14.1165</td>
<td>14.5923</td>
<td>16.5408</td>
</tr>
<tr>
<td>Sub-channel 7</td>
<td>15.1109</td>
<td>13.6386</td>
<td>13.1869</td>
<td>0</td>
</tr>
<tr>
<td>Sub-channel 8</td>
<td>12.0235</td>
<td>14.7640</td>
<td>15.2337</td>
<td>12.5081</td>
</tr>
</tbody>
</table>

TABLE IV. Rates of D2D pairs and sum rate values (bit/s/Hz) for convexified and non-convex problems (NBS game).

<table>
<thead>
<tr>
<th></th>
<th>Rate (Convexified)</th>
<th>Rate (Non-convex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2D 1</td>
<td>16.7590</td>
<td>16.7590</td>
</tr>
<tr>
<td>D2D 2</td>
<td>18.7484</td>
<td>18.7484</td>
</tr>
<tr>
<td>D2D 3</td>
<td>17.6002</td>
<td>17.6002</td>
</tr>
<tr>
<td>D2D 4</td>
<td>16.1742</td>
<td>16.1742</td>
</tr>
<tr>
<td>Sum</td>
<td>69.2818</td>
<td>69.2819</td>
</tr>
</tbody>
</table>

Fig. 3. Powers of D2D pairs versus the number of iterations for Nash Competitive game. The powers values have been converged after 12 iterations.

Fig. 4. Rates of D2D pairs versus the number of iterations for Nash Competitive game. The rates have been converged after 12 iterations.

do not change after a threshold value for $I_{ij}$. They converge to various specific values which depend on the $P_{max}$ we used for each of them. This states that C3 constraint has been relaxed and inactive and C1 constraint would become important and has direct influence on the powers and rates of D2D pairs. This figure also demonstrates that D2D pairs have higher rates in NBS game compared to Nash Competitive game which we had concluded from comparison between Table IV values and Fig. 4 too.

V. CONCLUSION

Considering the fact that underlaying D2D communication causes interference on the traditional cellular communication, this paper proposed NBS and Nash Competitive games to implement power allocation for D2D underlay cellular network. The utility function of each UE is modeled by its rate. We analyzed the optimal solutions for the constructed games. Since NBS game problem was not convex, we proposed a method to make it convex and then we found NBS. In contrast, Nash Competitive problem was convex. Thus, we applied KKT conditions and derived a closed form solution for NE. Simulation results indicate that the power and rate values for D2D pairs are 99 percent similar to each other for non-convex and our convexified problems for NBS game. This confirms that our approach is exact and works well. Furthermore, our method works well for different ranges of SINR. Besides, the results show that D2D pairs achieve higher rates in NBS game than in Nash Competitive game. This illustrates that D2D pairs cooperation provides them more efficiency.

APPENDIX A

PROOF OF THE THEOREM 1

In order to show that the problem is convex, we prove that the objective function is concave with respect to $\tilde{p}_{ij}$ and the constraints are convex sets. Since the constraints are linear combinations of exponential terms of $\tilde{p}_{ij}$, they are convex sets [17]. In order to prove that the objective function is concave with respect to $\tilde{p}_{ij}$, we should calculate the Hessian matrix for it. If the Hessian matrix is negative semidefinite, the function will be concave [17]. Here, we find the Hessian matrix for the case we have two D2D users and one sub-channel. The extension to $D$ users and $N$ sub-channels is straightforward and omitted because of brevity.
The rate of each D2D user is as follows,

\[
U_1(p_1, p_2) = \log_2 \left( 1 + \frac{g_{11}p_1}{\sigma_1^2 + g_{12}p_2 + P_{\text{inc}}} \right), \tag{19}
\]

\[
U_2(p_1, p_2) = \log_2 \left( 1 + \frac{g_{22}p_2}{\sigma_2^2 + g_{21}p_1 + P_{\text{inc}}} \right). \tag{20}
\]

By applying (6) and changing the variable \( p_i \) with \( e^{\bar{p}_i} \), we have the following formulas for the rate of each D2D user:

\[
U_1(p_1, p_2) = \alpha_1 \log_2 \left( 1 + \frac{g_{11}e^{\bar{p}_1}}{\sigma_1^2 + g_{12}e^{\bar{p}_2} + P_{\text{inc}}} \right) + \beta_1, \tag{21}
\]

\[
U_2(p_1, p_2) = \alpha_2 \log_2 \left( 1 + \frac{g_{22}e^{\bar{p}_2}}{\sigma_2^2 + g_{21}e^{\bar{p}_1} + P_{\text{inc}}} \right) + \beta_2. \tag{22}
\]

The Hessian matrices for D2D users are calculated as follows,

\[
H_1 = \begin{bmatrix}
\frac{\partial^2 U_1}{\partial^2 p_1} & \frac{\partial^2 U_1}{\partial p_1 \partial p_2} \\
\frac{\partial^2 U_1}{\partial p_2 \partial p_1} & \frac{\partial^2 U_1}{\partial^2 p_2}
\end{bmatrix} = \begin{bmatrix}
\frac{0}{0} & -\frac{\alpha_1}{\sigma_1^2} \times \frac{g_{12}e^{\bar{p}_2} (\sigma_1^2 + g_{12}e^{\bar{p}_2} + P_{\text{inc}})}{\left(\sigma_1^2 + g_{12}e^{\bar{p}_2} + P_{\text{inc}}\right)^2} \\
\end{bmatrix} \tag{23}
\]

\[
H_2 = \begin{bmatrix}
\frac{\partial^2 U_2}{\partial^2 p_2} & \frac{\partial^2 U_2}{\partial p_2 \partial p_1} \\
\frac{\partial^2 U_2}{\partial p_1 \partial p_2} & \frac{\partial^2 U_2}{\partial^2 p_1}
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{\alpha_2}{\sigma_2^2} \times \frac{g_{21}e^{\bar{p}_1} (\sigma_2^2 + g_{21}e^{\bar{p}_1} + P_{\text{inc}})}{\left(\sigma_2^2 + g_{21}e^{\bar{p}_1} + P_{\text{inc}}\right)^2} \\
\end{bmatrix} \tag{24}
\]

Since, three elements of the Hessian Matrices are equal to zero and the forth element is negative, these matrices are negative semidefinite, which indicates:

\[
H_1 \preceq 0, \quad H_2 \preceq 0. \tag{25}
\]

Thus, each of the D2D rates are concave. Therefore, the proof is completed.

REFERENCES


