Adaptive Rate and Power Allocation Schemes for OFDM/SDMA Systems

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Abstract

In this paper, adaptive resource allocation approaches, which jointly adapts power distribution and subcarrier allocation according to instantaneous channel conditions, is proposed for uplink OFDM/SDMA-based multiuser system. The first scheme utilizes adaptive algorithm to reduce symbol-error rate. The closed-form algorithm is derived for this idea. The second contribution in this paper deals with constant-rate transmissions over a spatial multiuser OFDM system. In the proposed system, the receiver jointly optimizes rate and power assignments for each user and then informs them to the transmitter via a low-rate feedback channels.

1. Introduction

Orthogonal frequency-division multiplexing (OFDM) has been considered as a very promising technique for data transmission in a number of environments, and is the main interface solution to many wireless systems [1]. OFDM has been adopted in PHY of some popular standards such as HIPERLAN2 and IEEE 802.11a. On the other hand, because of spectrum limitation, high spectral efficiency is crucial.

Higher data rates can be achieved by incorporating space division multiple access (SDMA), which makes bandwidth reuse possible by multiplexing signals based on their spatial signatures. Multiple antennas when deployed at both the transmitter and the receiver show an impressive gain in capacity of a wireless channel [2], [3]. Thus, OFDM can be combined with SDMA to achieve high data rates and at the same time limit the errors induced by adverse channel conditions to a minimum.

Although almost many works have done in adaptive resource allocation in downlink SDMA/OFDM systems (see, e.g., [4]-[5]), there are a few works for its uplink counterpart. In [6] and [7], an adaptive resource-management algorithm for multiuser transmission with OFDM signaling in uplink mode was presented, when both transmitter and receivers exploit multiple antennas. Since the usage of multiple antenna for user nodes are infeasible in most of applications, we consider a single antenna for each user. In addition, objective in [6] and [7] is transmit power minimization, given each user fulfills QoS requirements including bit-error rate and data rate.

In this paper, we are first deals with BER reduction providing each user guarantee the fulfillment of QoS requirements including transmitted power and data rate. In order to lower the BER of the detectors, we investigate the application of adaptive loading to the uplink OFDM/SDMA system. Previously, it was shown that adaptive loading also forms a viable technique for lowering the BER in a single-user OFDM [8]. In this paper, the adaptive loading technique based on [8] is extended to a multiuser OFDM/SDMA system, and it is shown to be an effective mean to improve the performance while processing a limited complexity.

In this paper, we also consider constant-rate transmission through uplink OFDM/SDMA. A useful measure of the block-fading channel capacity for transmissions of delay-sensitive data is the outage capacity, defined as the maximum constant-rate that can be reliably transmitted over all fading blocks under an average transmit power constraint [9]. In [10], Chung et al. proposed closed-loop V-BLAST architecture for MIMO channels to increase the achievable rate by jointly optimizing power and rate assignments for each transmit antenna based on the channel knowledge at the receiver and then feeding them back to the transmitter. It was shown in [10] that achievable rate of this channel capacity attained through eigenmode transmission. In this paper, we extend upon the work [10] by applying the closed-loop V-BLAST architecture to multiuser system with constant-rate transmissions and present the associated power and rate allocation scheme.

2. System Model

Consider a multiuser OFDM system with $U$ single transmit antenna users, a base station with $A$ receiver antennas and $N$ orthogonal tones. The transmission is assumed to be on a time-block basis and the wireless channel is assumed to be block-fading. A conventional channel with the baseband representation of the multipath channel from user $k$ to one of $A$ antennas. It is assumed that the cyclic prefix is long enough to suppress the intersymbol-interference (ISI). By applying standard OFDM
modulation and demodulation at the transmitter and the receiver, respectively, the demodulated signal during one transmission block of interest can be expressed as

\[ y[n] = \sum_{k=1}^{K} H^k[n] \sqrt{p^k[n]} x^k[n] + w[n] \]

\[ = \mathbf{H}[n] \mathbf{P}^{U2}[n] \mathbf{x}[n] + \mathbf{w}[n] \]

\[ \mathbf{P}^{U2}[n] = \text{diag}\left(\sqrt{p^1[n]}, \sqrt{p^2[n]}, \ldots, \sqrt{p^U[n]}\right), \]

\[ \mathbf{x}[n] = [x^1[n], x^2[n], \ldots, x^U[n]]^T, \]

where \( y[n] \) (4-dimensional) and \( \mathbf{x}[n] \) (4-dimensional) are the received and transmitted signal vectors at the \( n \)th OFDM tone, respectively. \( \mathbf{H}[n] (A \times U\text{-dimensional}) \) is the frequency-domain channel matrix at the \( n \)th tone, and \( \mathbf{H}^k[n] \) denote the \( k \)th column of \( \mathbf{H}[n] \). \( \mathbf{w}[n] \) (4-dimensional) is the additive noise at the receiver and it is assumed that \( \mathbf{w}[n] \sim \mathcal{CN}(0,1) \). \( \mathbf{P}^{U2}[n] \) is a matrix containing the power control coefficients \( \sqrt{p^k[n]} \).

Assume that user \( k \) has a data rate requirement of \( R_k \) bits per OFDM symbol. In each symbol duration, a data stream composed of \( R_k \) bits is fed into a bit-distribution block, which segments the data stream into \( N \) parallel streams with each containing \( R_k[1], R_k[2], \ldots, R_k[N] \) bits. These data streams are modulated into a symbol sequence \( x^k[1], x^k[2], \ldots, x^k[N] \) to be transmitted on the \( N \) tones. \( x^k[n] \) is then scaled according to its transmit power level \( p^k[n] \). In the proposed system, the values of \( R_k[n] \) and \( p^k[n] \) are controlled by the adaptive-allocation algorithm, which adapts these parameters according to the instantaneous channel conditions.

The receiver attempts to obtain estimates of the transmitted symbols \( \hat{x}^k[n], \hat{x}^k[1], \ldots, \hat{x}^k[N] \), which are denoted by \( \mathbf{x}^k[n], \mathbf{x}^k[1], \ldots, \mathbf{x}^k[N] \), from the received signals \( y[n] \) for all subcarriers \( n \). At the receiver, each data stream can then be decoded by various multiluser detection techniques known for SDMA (see, e.g. [11].

3. SER Reduction by Rate and Power Control

Our objective in this section is to minimize the symbol-error rate (SER) by setting the appropriate rate and transmit power levels. It can be easily shown that using the Gaussian approximation of multiluser interference model, minimizing the SER is equivalent to maximizing the SINR. For M-QAM constellations, we have [13]

\[ \text{SER}^k[n] \approx 4Q \left( \frac{3R^k[n]}{p^k[n]E^k[n]} \right)^{1/2} \]

where \( E^k[n] \) and \( N^k[n] \) are the average power per bit and Gaussian approximated noise plus interference of the \( n \)th tone of the user \( k \), respectively. \( \text{SER}^k[n] \) is the SER of the \( n \)th tone of the user \( k \).

On the other hand, it can be shown (see for example [13]) that the symbol error probability of the \( n \)th tone of the user \( k \) is presented by

\[ \text{SER}^k[n] \approx 4Q \left( \frac{2p^k[n]}{N^k[n]} \right)^{1/2} \]

Comparing (4) and (5), the transmit power \( S^k[n] \) for \( n \)th tone of the user \( k \) can be written as

\[ S^k[n] = p^k[n] \left( 2^{R^k[n]} - 1 \right) \approx \frac{2}{3} p^k[n] 2^{R^k[n]} \]

For simplicity and to reduce the amount of feedback, we constraint that the term \( N^k[n] \) becomes independent of other users expressions in network. Thus, Zero-Forcing (ZF) multiluser detection could be an appropriate selection, since it eliminates completely the interference from the other users. Then,

\[ N^k[n] = N_0 \left| \mathbf{F}^k[n] \right|^2, \]

where \( \mathbf{F}^k[n] \) is the \( k \)th row of \( A \)-dimensional ZF weight matrix \( \mathbf{F}[n] \) which is the pseudoinverse of the channel matrix. It is obvious that in the optimum system design all subcarriers perform with the same error rate. Otherwise, the highest error rate would dominate. Thus, we demand the same SER for all subcarriers, which together with (5), translate to SNR independent of the index \( n \). Therefore, for minimizing the SER, the following SNR could be maximized

\[ \text{SNR} = \frac{p^k[n]}{N_0 \left| \mathbf{F}^k[n] \right|^2} \]

Furthermore, in the proposed system, each user can have its own rate and power requirements, which can be represented by

\[ R^k = \sum_{n=1}^{N} R^k[n], S^k = \sum_{n=1}^{N} S^k[n] \]

Solution to the constraint optimization problem (8)-(9) can be derived by the same approach as [8].
\[ R^k[n] = \frac{R^k}{N} + \frac{R^k}{N} \log \left( \prod_{i=1}^{N} \frac{|F^i[n]|^2}{\|F^i[n]\|^{2N}} \right), \quad (10) \]

\[ p^k[n] = \frac{3S^k}{2^{N+1}} \sum_{i=1}^{N} \frac{|F^i[n]|^2}{\|F^i[n]\|^{2N}}. \quad (11) \]

Thus, closed-form solution for rate and power are obtained which indicate that we can feed back either the projection vectors \( F^k[n] \) or rate and power coefficients \( R^k[n] \) and \( p^k[n] \) from the base station.

Since, for high SNR scenarios, MMSE multiuser detection technique has asymptotically the same structure as ZF detector, we apply this adaptive loading also for the case of MMSE detection. The performance improvement using the proposed adaptive method is investigated in Section V.

4. Power Optimization Using Rate and Power Feedback

In this section, like section III, we are going to adaptively allocate rate and power to different tones, considering data rate requirement of \( R^k \) bits per OFDM symbol. So system model presented in Section II are again applicable for this scenario. But, instead of the problem in the previous section which minimized the received power, we are going to minimize the transmitted power from users given a rate constraint. Furthermore, unlike the previous problem which uses the SER as an optimization metric, we take information theoretic approach for this latter problem.

The transmitted signal at each tone, \( s[n] = P^k[n] x[n] \), is assumed to be Gaussian vector and \( s[n] \sim \mathcal{CN}(0, \Sigma[n]) \).

At the receiver, it is assumed zero-mean complex Gaussian noise with unit variance is added to each antenna. The maximum transmission rate (in bits/sec/Hz) at each transmission block can be obtained from the mutual information between the channel input and output as

\[ R = \frac{1}{2N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{\|H[n]\Sigma[n]H[n]\|^2}{\|N[n]\|^2} \right) \], \quad (12)

where \( (.)^* \) denotes he conjugate transpose of the matrix.

In this section, we consider outage capacity with a rate \( R \) and the objective is to minimize the required transmit power per block denoted by

\[ P = \frac{1}{N} \sum_{n=1}^{N} \text{Tr}(\Sigma[n]) \] \quad (13)

The optimal structure of \( \Sigma[n] \) to minimize the power required to support rate \( R \) in (12) is based on the singular-value decomposition (SVD) of the channel matrix [14], \( H[n] \), i.e., \( H[n] = U[n] \Lambda^{1/2}[n] V[n] \), where \( U[n] \) and \( V[n] \) are \( A \times M \) and \( M \times U \) dimensional matrices, respectively, which satisfy \( U^H[n] U[n] = I \) and \( V^H[n] V[n] = I \), respectively, \( M = \min(U, A) \), and \( \Lambda^{1/2}[n] = \text{diag}(\lambda_1[n], \lambda_2[n], ..., \lambda_M[n]) \) with \( \lambda_i[n] \geq 0 \), \( i = 1, ..., M \). Note that the transmit signal covariance, \( \Sigma[n] \) can take the form of \( V^H[n] \Gamma_n V[n] \), where \( \Gamma_n = \text{diag}(p^1[n], p^2[n], ..., p^M[n]) \) and \( p^k[n] \) denotes the power allocation at the \( k \)th subchannel, where if we suppose \( U \leq A \), \( p^k[n] \) will be the \( k \)th user power allocation. Our task is to find the optimal values of \( \{ p^k[n] \} \) such that the average power for each block is minimized and transmission rate per block is no less than the target rate \( R \). This can be achieved by solving the following optimization problem

**Minimize** \( P = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{U} p^k[n] \) \quad (14)

**s.t.** \( \frac{1}{2N} \sum_{n=1}^{N} \sum_{k=1}^{U} \log_2 \left( 1 + \frac{\lambda_k[n]}{\lambda_k[n]} p^k[n] \right) \geq R \). \quad (15)

The above problem can be easily solved by the “water-filling” algorithm [14], which assigns the powers according to \( p^k[n] = (\mu - 1/\lambda_k[n])^+ \), where \( \mu (\geq 0) \) is the unique “water-level” under which \( (1/2N) \sum_{n=1}^{N} \sum_{k=1}^{U} \log_2 (\mu + \lambda_k[n])^+ = R \).

Now, we extend the closed-loop V-BLAST architecture proposed in [10] for MIMO channels to a Multiuser OFDM system that has \( U \) users transmitting with single antenna over \( N \) tones to the base station with \( A \) receive antennas. We assume that the receiver has full knowledge of the channel at each fading block while the transmitter has no such knowledge. Based on the channel knowledge, the receiver determines a set of power and rate assignments for each transmit user and then informs them to the transmitters via a reliable feedback channel. It is assumed that the feedback delay is negligible compared to the fading rate and so the channel is not changed during the feedback. As the representation in Section II we assume each user is carrying a portion of the total bits that is equal to \( R_k/\beta, k = 1, ..., M \), each data stream is independently processed and the output symbols are then power amplified. The power amplification for the \( k \)th data streams is based on the power vector
\( \{p^k[1], p^k[2], \ldots, p^k[N]\} \) and \( p^k[n] \) controls the power of the symbols assigned to the \( k \)th OFDM tone. At the receiver, each data stream can be then decoded by various multiuser detection techniques known for SDMA (see, e.g., [11]-[12]).

First, we are going to utilize ZF method to decode the data stream from each user. We can show that the equivalent channel for decoding the \( k \)th data stream can be expressed as

\[
\tilde{x}^k[n] = \sqrt{p^k[n]x^k[n]} + \left| \mathbf{F}^k[n] \right| v^k[n],
\]

where \( v^k[n] \sim \mathcal{CN}(0,1) \) and \( \left| \mathbf{F}^k[n] \right| \) is the \( k \)th row of \( U \times A \)-dimensional ZF weight matrix \( \mathbf{F}[n] \) which is the pseudoinverse of the channel matrix. From (16), the maximum rate achievable for each user can be obtained as

\[
R^k = \frac{1}{2N} \sum_{n=1}^{N} \log \left( 1 + \frac{p^k[n]}{\left| \mathbf{F}^k[n] \right|^2} \right),
\]

Like in [10], we also employ the Optimum Successive Decoding (OSD) in this paper to decode the data stream from each user. OSD is equivalent to minimum mean squared-error generalized decision-feedback equalization (MMSE-GDFE) for Gaussian ISI channels [10]. OSD is parameterized by a set of \( A \)-dimensional projection vectors \( \mathbf{F}^k_{\text{OSD}}[n] \), and detection vectors which are obtained by subtracting the decoded signals associated with users \( 1 \) to \( k \) from the received signal. So, estimated signals from the \( k \)th user as \( \hat{x}^k[n] \) can be obtained as

\[
\hat{x}^k[n] = \mathbf{F}^k_{\text{OSD}}[n] \left( \mathbf{y}[n] - \sum_{j=k+1}^{U} x^j[n] \mathbf{H}^j[n] \right),
\]

The projection vector takes into account the interference signals from not decoded users \( k+1 \) to \( U \) and minimizes the mean squared-error in \( \hat{x}^k[n] \):

\[
\begin{align*}
\mathbf{F}^k_{\text{OSD}}[n] &= \left[ \sum_{j=k+1}^{U} x^j[n] \mathbf{H}^j[n] p^j[n] \mathbf{H}^j[n]^H \right]^{-1} \mathbf{H}^k[n],
\end{align*}
\]

It is not hard to show that the equivalent channel for decoding the \( k \)th data stream can be now expressed as

\[
\tilde{x}^k[n] = \sqrt{p^k[n]} x^k[n] + v^k[n],
\]

where \( v^k[n] \sim \mathcal{CN}(0,1) \) denote the equivalent noise. From (20), the maximum rate achievable for each user can be obtained as

\[
R^k = \frac{1}{2N} \sum_{n=1}^{N} \log \left( 1 + \mathbf{F}^k_{\text{OSD}}[n]^H \mathbf{H}^k[n] p^k[n] \right),
\]

For determining the power allocation at each user as well as at each OFDM tone, our objective is to minimize the power per block, \( P \), given that the sum-rate from all users is no less than the target rate \( R \). From (13), it can be shown that the associated optimization problem is given as below:

\[
\begin{align*}
\text{Minimize} & \quad P = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{U} p^k[n] \\
\text{s.t.} & \quad \frac{1}{2N} \sum_{n=1}^{N} \log \left( 1 + \sum_{k=1}^{U} \mathbf{F}^k[n]^H \mathbf{F}^k[n] \right) \geq R.
\end{align*}
\]

The above problem is convex and therefore can be solved by using standard convex optimization techniques. It can be derived that the optimal values for \( \{p^k[n]\} \) at tone \( n \) need to satisfy

\[
\begin{align*}
p^k[n] &= \left( \frac{\mu}{2 \log 2} - \frac{1}{\beta^k[n]} \right)^+ , \quad k = 1, \ldots, U ,
\end{align*}
\]

where \( \beta^k[n] \) is

\[
\begin{align*}
\beta^k[n] &= \left| \mathbf{F}^k_{\text{OSD}}[n]^H \mathbf{H}^k[n] \right| \\
\beta^k[n] &= \frac{1}{\left| \mathbf{F}^k[n] \right|^2},
\end{align*}
\]

for the case of OSD and ZF equalization, respectively. Note that (24) can be obtained by iteratively computing \( \{\beta^k[n]\} \) and \( \{p^k[n]\} \) until both converge. The proof of such convergence is similar to that for the iterative water-filling algorithm in [15].

5. Simulation Results

For investigating the performance of the proposed adaptive OFDM/SDMA system, we employ a new IEEE 802.11a simulation model [11], [12]. In the IEEE 802.11a-based uncoded OFDM/SDMA uplink transmission scenario, each of \( U \) simultaneous users is equipped with one antenna, while the receiver is equipped with \( A = 4 \) antennas to exploit spatial diversity. For each user, the random source data bits first are mapped to QAM symbols. Then OFDM transmitter modulates them on correspondence sub-carriers by \( N = 64 \) point. To eliminate ISI, each data OFDM symbol is preceded by a cyclic prefix which contains the last \( N_c = 16 \) tones of the data OFDM symbol.

The performance for MMSE detector with and without adaptive loading, in terms of the average bit error rate (BER) as a function of the received SNR per symbol and per antenna is shown in Fig. 1. The adaptive procedure described in Section III. In the case where no adaptive
loading of the tones is performed, this implies that QPSK is applied on each tone. For adaptive loading, we will use QPSK, 16QAM, 64QAM or no modulation on the tones, which are the supported constellation size in IEEE 802.11a. No channel coding is included in our results, as we are interested in the gains of adding adaptive loading.

The curves in Fig. 1 show the BER versus SNR for $U=1,4$, the solid lines represent the standard MMSE uplink, and the dashed lines represent the performance of the two loading algorithms. Clearly, adaptive loading is able to effectively exploit the available frequency diversity. For $U=4$, the increased diversity order results in a gain of 8 dB for a BER of $10^{-3}$. For lower $U$, like $U=1$, the gains due to adaptive loading diminish, since MMSE detection provides an order of diversity $A-U+1$.

Fig. 2 shows the achievable rates versus the average SNR per receive antenna based on the discussions in Section IV. The upper limit bound as performance criteria is depicted based on optimization problem (15)+(16). It is observed that the proposed power and rate allocations algorithm based on the OSD method outperforms the one based on ZF equalizer. Moreover, Fig. 2 demonstrates that the method based on ZF have better performance than the case with equal power allocation in all users and tones.

6. References


