Impact of Transceiver I/Q Imbalance on Transmit Diversity of Beamforming OFDM Systems

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***Abstract*— One of the serious imperfections affecting OFDM systems is transceiver I/Q imbalance. In this letter, closed-form expressions for the outage probability of beamforming OFDM systems with transmit and receive I/Q imbalances are derived. Moreover, the asymptotic behavior and diversity order of the system is investigated. The analytical results are confirmed by simulations.**

***Index Terms*— OFDM, I/Q imbalance, performance analysis, beamforming, asymptotic analysis**

I. Introduction

Multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems, involving clever processing in both the spatial and frequency domains, have been proposed for WiFi, WiMax, and fourth generation cellular systems as well as the IEEE 802.16 standard for wireless Internet access. A low-cost implementation of such physical layers is desirable in view of mass deployment, but challenging due to impairments associated with the analog components. A major source of analog impairments in high-speed wireless communications systems is the in-phase/quadrature-phase (I/Q) imbalance [1], [2]. The I/Q imbalance is the mismatch between I and Q balances due to the analog imperfection and introduced both in the up and downconversion at the transceivers. In general, it is difficult to efficiently and entirely eliminate such imbalances in the analog domain due to power consumption, size, and cost of the devices. Therefore, efficient compensation techniques in the digital baseband domain are needed for the transceivers [3]–[6].

This letter provides an analysis of the outage probability of the maximum ratio transmission (MRT) based OFDM system with transceiver I/Q imbalance of low cost terminals. The closed-form expressions for the outage probability of the beamforming OFDM system with either transmit or receive I/Q imbalances are derived. It is shown that in a system with transmit I/Q imbalance, spatial diversity is still achievable. It is shown that in a system with transmit I/Q imbalance, full spatial diversity is still achievable, if the condition $A_T < (2^{R_k}-1)^{-1}$ is satisfied where R_k is the transmit rate and A_T is the image-leakage-ratio of the transmit I/Q imbalance. It is demonstrated that the system with receive I/Q imbalance becomes interference-limited, and the error-floor occurred in high SNR scenarios is found. Furthermore, an approximated formula for the system which suffers from both transmit and receive I/Q imbalances is obtained.

The rest of this letter is organized as follows: Section II reviews the model of a beamforming MIMO OFDM system with transceiver I/Q imbalance. This model is statistical and builds the basis for the analysis of the system outage and diversity order in Section III. In Section IV, the overall system performance is presented via simulations and the correctness of the analytical formulas is confirmed by simulation. Conclusions are drawn in Section V.

II. System Model

We consider a MIMO OFDM beamforming system transmitting over K subcarriers with M antennas at the transmitter and a single antenna at the receiver. The system wants to transmit data symbol s_k on the k-th subcarrier for $k \in \{-K/2, -K/2+1, \ldots, K/2\}$, where s_k is taken from some two-dimensional symbol constellation. We assume that no data is transmitted on the DC subcarrier and K is an even number. It is assumed that each subcarrier symbol is transmitted with equal average energy E_s. At the transmitter, the k-th subcarrier is modulated by the symbol s_k using the $M \times 1$ beamforming vector $\boldsymbol{w}_k$. We set $\|\boldsymbol{w}_k\| = 1$, to reflect the power constraint at the transmitter, where $\|\cdot\|$ denotes the Euclidian norm. Then, varying the beamforming vector, the maximum SNR is achieved if $\boldsymbol{w}_k$ is proportional to $\boldsymbol{h}_k^*$ where $(\cdot)^*$ represents the complex conjugate, and $\boldsymbol{h}_k$ is the k-th subcarrier channel vector with Gaussian distribution due to Rayleigh fading. This transmission scheme is commonly described as MRT, which achieves full diversity and the full array gain in Rayleigh fading channels [7], [8]. Thus, we have

$$\boldsymbol{w}_k = \frac{\boldsymbol{h}_k^*}{\|\boldsymbol{h}_k\|}. \tag{1}$$

It is assumed that the impulse response of the channel is shorter than the cyclic prefix. After removing the cyclic prefix, the channel for the k-th subcarrier after the Discrete Fourier Transform (DFT) can be described as a $M \times 1$ complex channel vector $\boldsymbol{h}_k$. Let $\boldsymbol{x}_k$ be a $M \times 1$ complex valued vector that is

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transmitted at subcarrier k. Then, the received signal on the k-th subcarrier can be written as

$$y_k = \boldsymbol{h}_k^t \boldsymbol{x}_k + n_k, \tag{2}$$

where $(\cdot)^t$ denotes the transpose, n_k is the zero-mean noise with independent and identically distributed (i.i.d.) circular symmetric Gaussian distribution and variance $\mathcal{N}_0$. Note that each $\boldsymbol{x}_k$ may differ from the original signal $\boldsymbol{w}_k s_k$ at subcarrier k. Only in case of an ideal transmitter $\boldsymbol{x}_k = \boldsymbol{w}_k s_k$.

In [9], [10], it has been shown that I/Q imbalance of a direct-conversion transceiver in OFDM systems induces a mutual interference between subcarriers that are located symmetrically to the DC carrier. Since there is a difference between the impact of I/Q imbalance at the receiver side and I/Q imbalance at the transmitter side, it is reasonable to model the transmit and receive I/Q imbalances separately.

A. Transmit I/Q Imbalance

Before transmission, the baseband signal is up-converted to the radio frequency signal by using a local oscillator (LO) of the carrier frequency. Ideally, the LO outputs for the I and Q branches (representing the real part and imaginary parts) should have equal amplitudes and phase difference of $\pi/2$. However, in practice, the matching of I and Q signals is imperfect and this leads to the amplitude and phase imbalance between the I and Q signals. Such impairments are known as Tx IQ imbalance or mismatch and this severely limits the performance of the receiver, especially if cheap components or architectures, e.g., direct conversion architecture [11], are employed. The IQ distorted signal of the kth subcarrier can be modeled using the original signal and its conjugate as

$$\boldsymbol{x}_k = \alpha_{\mathrm{T}} \boldsymbol{w}_k s_k + \beta_{\mathrm{T}} \boldsymbol{w}_{-k}^* s_{-k}^*, \tag{3}$$

where two complex scalers α_{T} and β_{T} are given by [1], [2]

$$\begin{aligned} \alpha_{\mathrm{T}} &= \cos\theta_{\mathrm{T}} + \jmath\, \epsilon_{\mathrm{T}} \sin\theta_{\mathrm{T}}, \\ \beta_{\mathrm{T}} &= \epsilon_{\mathrm{T}} \cos\theta_{\mathrm{T}} - \jmath \sin\theta_{\mathrm{T}}, \end{aligned} \tag{4}$$

where ϵ_{T} and θ_{T} denote the amplitude and phase imbalances between the I and Q branches of the transmitted signal, respectively. When the matching of I and Q balances is ideal, i.e., $\epsilon_{\mathrm{T}} = 0$ and $\theta_{\mathrm{T}} = 0$, then $\alpha_{\mathrm{T}} = 1$ and $\beta_{\mathrm{T}} = 0$. The degree of the I/Q imbalance can be evaluated in terms of a image-leakage-ratio, which is defined by

$$A_{\mathrm{T}} \triangleq \frac{|\beta_{\mathrm{T}}|^2}{|\alpha_{\mathrm{T}}|^2}.$$

From (1)-(3), we have

$$y_k = \alpha_{\mathrm{T}} \|\boldsymbol{h}_k\| s_k + \beta_{\mathrm{T}} \frac{\boldsymbol{h}_{-k}^\dagger \boldsymbol{h}_k}{\|\boldsymbol{h}_{-k}\|} s_{-k}^* + n_k, \tag{5}$$

where $(\cdot)^\dagger$ represents the conjugate transpose. As can be seen from (5), the received signal results from a superposition of the transmitted symbols s_k, the interfering transmitted symbol s_{-k}^*, and noise samples. Hence, the signal-to-interference-and-noise- ratio (SINR) at the k-th subcarrier can be written as

$$\begin{aligned} \mathrm{SINR}_k^{\mathrm{T}} &= \frac{E_s\,|\alpha_{\mathrm{T}}|^2 \sum_{m=1}^M |h_{k,m}|^2}{E_s |\beta_{\mathrm{T}}|^2 \frac{|\sum_{m=1}^M h_{k,m} h_{-k,m}^*|^2}{\sum_{m=1}^M |h_{-k,m}|^2} + \mathcal{N}_0} \\ &\geq \frac{E_s\,|\alpha_{\mathrm{T}}|^2 \sum_{m=1}^M |h_{k,m}|^2}{E_s |\beta_{\mathrm{T}}|^2 \sum_{m=1}^M |h_{k,m}|^2 + \mathcal{N}_0}, \end{aligned} \tag{6}$$

where $h_{k,m}$ and $h_{-k,m}$ are the m-th component of the channel vectors $\boldsymbol{h}_k$ and $\boldsymbol{h}_{-k}$, respectively. The inequality in (6) holds using Cauchy-Schwarz inequality, i.e., $\|\boldsymbol{h}_k \boldsymbol{h}_{-k}\| \leq \|\boldsymbol{h}_k\|\,\|\boldsymbol{h}_{-k}\|$.

B. Receive I/Q Imbalance

Consider the multiple transmit-antenna OFDM system with I/Q imbalance at the receiver front-end. Because of the I/Q imbalance, each receive symbol y_k at subcarrier k is interfered by the complex conjugated symbol y_{-k}^* received at subcarrier $-k$ and vice versa. Thus, the received signal can be modeled as

$$r_k = \alpha_{\mathrm{R}} y_k + \beta_{\mathrm{R}} y_{-k}^*, \tag{7}$$

where r_k denotes the actually received symbol at subcarrier k. The complex valued weighting factors α_{R} and β_{R} of the receiver front-end are defined as

$$\begin{aligned} \alpha_{\mathrm{R}} &= \cos\theta_{\mathrm{R}} + \jmath\, \epsilon_{\mathrm{R}} \sin\theta_{\mathrm{R}}, \\ \beta_{\mathrm{R}} &= \epsilon_{\mathrm{R}} \cos\theta_{\mathrm{R}} - \jmath \sin\theta_{\mathrm{R}}, \end{aligned} \tag{8}$$

where ϵ_{R} and θ_{R} denote the amplitude and phase imbalances between the I and Q branches of the received signal, respectively. The degree of the receive I/Q imbalance can be evaluated in terms of a image-leakage-ratio, which is defined by

$$A_{\mathrm{R}} \triangleq \frac{|\beta_{\mathrm{R}}|^2}{|\alpha_{\mathrm{R}}|^2}.$$

Assuming an ideal transmitter, i.e., $x_k = \boldsymbol{w}_k s_k$, and combining (2) and (7) yields

$$r_k = \alpha_{\mathrm{R}} \|\boldsymbol{h}_k\| s_k + \beta_{\mathrm{R}} \|\boldsymbol{h}_{-k}\| s_{-k}^* + \alpha_{\mathrm{R}} n_k + \beta_{\mathrm{R}} n_{-k}^*. \tag{9}$$

From (9), the received SINR at the k-th subcarrier can be written as

$$\mathrm{SINR}_k^{\mathrm{R}} = \frac{\sum_{m=1}^M |h_{k,m}|^2}{A_{\mathrm{R}} \sum_{m=1}^M |h_{-k,m}|^2 + \lambda \Lambda_0}, \tag{10}$$

where $\lambda_0 = (1 + A_{\mathrm{R}}) \frac{\mathcal{N}_0}{E_s}$.

C. Combined Transmit and Receive I/Q Imbalances

A similar approach can be used to model I/Q imbalances at both the transmitter and the receiver. The received baseband equivalent frequency domain signal is given by

$$\begin{aligned} r_k = &\left(\alpha_{\mathrm{T}} \alpha_{\mathrm{R}} \|\boldsymbol{h}_k\| + \beta_{\mathrm{T}}^* \beta_{\mathrm{R}} \frac{\boldsymbol{h}_{-k}^\dagger \boldsymbol{h}_k}{\|\boldsymbol{h}_k\|} \right) s_k + \\ &\left(\beta_{\mathrm{T}} \alpha_{\mathrm{R}} \frac{\boldsymbol{h}_{-k}^\dagger \boldsymbol{h}_k}{\|\boldsymbol{h}_{-k}\|} + \alpha_{\mathrm{T}}^* \beta_{\mathrm{R}} \|\boldsymbol{h}_{-k}\| \right) s_{-k}^* + \alpha_{\mathrm{R}} n_k + \beta_{\mathrm{R}} n_{-k}^*. \end{aligned} \tag{11}$$

Next, considering the superposition of the transmitted symbols s_k, the interfering transmitted symbol s_{-k}^*, and noise samples in (11), the instantaneous SINR at the k-th subcarrier can be calculated as

$$\text{SINR}_k^{\text{TR}} = \frac{\|\boldsymbol{h}_k\|^2 + A_\text{T}A_\text{R}\frac{\|\boldsymbol{h}_{-k}^\dagger\boldsymbol{h}_k\|^2}{\|\boldsymbol{h}_k\|^2}}{A_\text{T}\frac{\|\boldsymbol{h}_{-k}^\dagger\boldsymbol{h}_k\|^2}{\|\boldsymbol{h}_{-k}\|^2} + A_\text{R}\|\boldsymbol{h}_{-k}\|^2 + \lambda_\text{TR}}$$
$$\geq \frac{\sum_{m=1}^M |h_{k,m}|^2 + A_\text{T}A_\text{R}\frac{\|\boldsymbol{h}_{-k}^\dagger\boldsymbol{h}_k\|^2}{\|\boldsymbol{h}_k\|^2}}{A_\text{T}\sum_{m=1}^M |h_{k,m}|^2 + A_\text{R}\sum_{m=1}^M |h_{-k,m}|^2 + \lambda_\text{TR}}, \tag{12}$$

where $\lambda_\text{TR} = \frac{(1+A_\text{R})}{|\alpha_\text{T}|^2}\frac{\mathcal{N}_0}{E_s}$ and we have used Cauchy-Schwarz inequality, i.e., $\|\boldsymbol{h}_k\boldsymbol{h}_{-k}\| \leq \|\boldsymbol{h}_k\|\,\|\boldsymbol{h}_{-k}\|$ for deriving the inequality in (12). Assuming that the effect of I/Q imbalance is low, we have $A_\text{T}A_\text{R} \approx 0$. Therefore, the SINR in (12) can be simplified to

$$\text{SINR}_k^{\text{TR}} \geq \frac{\sum_{m=1}^M |h_{k,m}|^2}{A_\text{T}\sum_{m=1}^M |h_{k,m}|^2 + A_\text{R}\sum_{m=1}^M |h_{-k,m}|^2 + \lambda_\text{TR}}. \tag{13}$$

III. Outage Probability and Asymptotic Analysis

In the following, the outage probability $P_\text{out} \triangleq \Pr\{C_k < R_k\}$ of the k-th received signal is investigated, where R_k is the transmit rate and $C_k = \log_2(1+\text{SINR}_k)$ is the supported rate. This probability depends on the fixed transmission parameters, I/Q imbalance parameters, and the channel conditions.

A. Outage Due to Transmit I/Q Imbalance

Now, we consider the system with transmit I/Q imbalance, and we calculate a closed-form expression for outage probability at the k-th subcarrier. By defining $\gamma_\text{req} \triangleq (2^{R_k} - 1)$, from (6), the outage probability at the k-th subcarrier can be represented as

$$P_\text{out}^\text{T} \triangleq \Pr\left\{\text{SINR}_k^\text{T} < \gamma_\text{req}\right\}.$$

Lemma *1:* Consider a finite set of independent and identical random variables $\mathcal{X} = \{X_1, \ldots, X_M\}$ with exponential distribution and mean σ_x^2. The cumulative distribution function (CDF) of

$$\text{SINR} = \frac{\sum_{m=1}^M X_m}{1 + A\sum_{m=1}^M X_m}$$

can be calculated as

$$\Pr\{\text{SINR} < \gamma\} = 1 - u(1 - A\gamma) \times \sum_{n=0}^{M-1} \frac{\gamma^n}{\sigma_x^{2n}\, n!\,(1 - A\gamma)^n} e^{-\frac{\gamma}{\sigma_x^2(1-A\gamma)}}, \tag{14}$$

where $u(x) = 1$ for $x > 1$ and $u(x) = 0$ for $x \leqslant 0$.

Proof: We define $X = \sum_{m=1}^M X_m$ which has an Erlang distribution M degrees of freedom with probability density function (PDF) $p_x(x) = \frac{x^{M-1}}{\sigma_x^{2M}(M-1)!}e^{-\frac{x}{\sigma_x^2}}$ [12]. Then, the CDF of the $\text{SINR} = \frac{X}{1+AX}$ can be calculated as

$$\Pr\{\text{SINR} < \gamma\} = \Pr\left\{\frac{X}{1+AX} < \gamma\right\}$$
$$= \begin{cases} \Pr\left\{X < \frac{\gamma}{1-A\gamma}\right\}, & \text{if } \gamma < \frac{1}{A}, \\ 1, & \text{if } \gamma \geq \frac{1}{A}, \end{cases}$$
$$= \begin{cases} 1 - \sum_{n=0}^{M-1} \frac{\gamma^n}{\sigma_x^{2n}\, n!\,(1-A\gamma)^n} e^{-\frac{\gamma}{\sigma_x^2(1-A\gamma)}}, & \text{if } \gamma < \frac{1}{A}, \\ 1, & \text{if } \gamma \geq \frac{1}{A}, \end{cases} \tag{15}$$

which is equivalent to (14). ■

From Lemma 1 and (6), and by defining $X_m \triangleq \frac{E_s|\alpha_\text{T}|^2|h_{k,m}^k|^2}{\mathcal{N}_0}$, $m = 1, \ldots, M$, when the condition $A_\text{T}\gamma_\text{req} < 1$, or $R_k < \log_2\left(1 + \frac{|\alpha_\text{T}|^2}{|\beta_\text{T}|^2}\right)$, is satisfied, the outage probability can be written as

$$P_\text{out}^\text{T} \leq 1 - \sum_{i=0}^{M-1} \frac{\gamma_\text{req}^i \mathcal{N}_0^i}{E_s^i|\alpha_\text{T}|^{2i}\sigma_{h_k}^{2i}\, i!\,(1 - A_\text{T}\gamma_\text{req})^i} \times e^{-\frac{\gamma_\text{req}\mathcal{N}_0}{E_s|\alpha_\text{T}|^2\sigma_{h_k}^2(1-A_\text{T}\gamma_\text{req})}} \triangleq P_\text{out}^\text{U}, \tag{16}$$

where $\sigma_{h_k}^2$ is the mean of the channel coefficients $|h_{k,m}|^2$, $m = 1, \ldots, M$. From (16), and by using the definition of diversity order $G_d = \lim_{\text{SNR}_s\to\infty} \frac{-\log(P_\text{out})}{\log(\text{SNR}_s)}$ [13, Eq. (1.19)], where $\text{SNR}_s = \frac{E_s}{\mathcal{N}_0}$, we have the following theorem.

Proposition *1:* For a beamforming MIMO OFDM system equipped with M transmit antennas and a single receive antenna, even in the existence of transmit I/Q imbalance, we can achieve the full spatial diversity order of M, when $R_k < \log_2\left(1 + \frac{|\alpha_\text{T}|^2}{|\beta_\text{T}|^2}\right)$.

Proof: We express the outage probability in (16) as $P_\text{out}^\text{U}(x) = 1 - e^{-x}\sum_{i=0}^{M-1}\frac{x^i}{i!}$, where $x = \frac{\gamma_\text{req}}{\text{SNR}_s|\alpha_\text{T}|^2\sigma_{h_k}^2(1-A_\text{T}\gamma_\text{req})}$. The n-th order derivatives of $P_\text{out}^\text{U}(x)$ at zero can be calculated as

$$P_\text{out}^{\text{U}(n)}(0) = 0,\ n = 0, 1, \ldots, M-1, \ \text{ and } \ P_\text{out}^{\text{U}(M)}(0) = 1. \tag{17}$$

Thus, the Taylor series of $P_\text{out}^\text{U}(x)$ can be expressed as $P_\text{out}^\text{U}(x) = \sum_{n=0}^\infty B_n x^n$, where $B_n = 0$, $n = 1, \ldots, M-1$, and $B_M = \frac{1}{M!}$. Hence, in high SNR scenario, the outage probability can be approximated as

$$P_\text{out}^\text{U} \approx \frac{\gamma_\text{req}^M}{\text{SNR}_s^M|\alpha_\text{T}|^2 M!\,\sigma_{h_k}^{2M}(1 - A_\text{T}\gamma_\text{req})^M}, \tag{18}$$

and by the fact that $x \to 0$ is equivalent to $\text{SNR}_s \to \infty$, we have $G_d = \lim_{\text{SNR}_s\to\infty}\frac{-\log(P_\text{out}^\text{U})}{\log(\text{SNR}_s)} = M$. Since an upper-bound on outage probability leads to the diversity order of M, it is obvious that the exact outage probability also achieves the full diversity order of M. Thus, the result given in Proposition 1 is obtained. ■

B. Outage Due to Receive I/Q Imbalance

Next, we consider the system with receive I/Q imbalance, and a closed-form expression for outage probability is calculated at the k-th subcarrier. From (10), the outage probability at the k-th subcarrier is defined as

$$P_\text{out}^\text{R} \triangleq \Pr\left\{\text{SINR}_k^\text{R} < \gamma_\text{req}\right\}.$$

Assuming that $\boldsymbol{h}_k$ and $\boldsymbol{h}_{-k}$ are uncorrelated, the random variables $X_m \triangleq \frac{|h_{k,m}|^2}{\lambda_0}$ and $Y_m \triangleq \frac{A_\text{R}|h_{-k,m}^k|^2}{\lambda_0}$, $m = 1, \ldots, M$ are independent exponential random variables with mean $\sigma_x^2 = \frac{\sigma_k^2}{\lambda_0}$ and $\sigma_y^2 = \frac{A_\text{R}\sigma_{-k}^2}{\lambda_0}$, respectively. Note that $\sigma_{h_k}^2$ and $\sigma_{h_{-k}}^2$ are the mean of the channel coefficients $|h_{k,m}|^2$ and $|h_{-k,m}|^2$, $m = 1, \ldots, M$, respectively. We define $Y \triangleq \sum_{m=1}^M Y_m$, which has an Erlang distribution with M degrees of freedom, hence, $p_y(y) = \frac{y^{M-1}}{\sigma_y^{2M}(M-1)!}e^{-\frac{y}{\sigma_y^2}}$. Moreover, $X=\sum_{n=1}^{M} X_n$ is Erlang distributed with CDF $\Pr\{X<x\} = 1-\sum_{n=0}^{M-1}\frac{x^n}{\sigma_x^{2n}\, n!}e^{-\frac{x}{\sigma_x^2}}$. By marginalizing over the random variable Y, we have

$$\Pr\left\{\mathrm{SINR}_k^{\mathrm{R}} < \gamma_{\mathrm{req}}\right\} = \int_0^\infty \Pr\left\{X < \gamma_{\mathrm{req}}(1+y)\right\} p_y(y)\, dy$$

$$= 1-\sum_{n=0}^{M-1}\int_0^\infty \frac{(\gamma_{\mathrm{req}}+\gamma_{\mathrm{req}}\, y)^n e^{-\frac{\gamma_{\mathrm{req}}(1+y)}{\sigma_x^2}}}{\sigma_x^{2n}\, n!}\frac{y^{M-1}e^{-\frac{y}{\sigma_y^2}}}{\sigma_y^{2M}(M-1)!}dy$$

$$= 1-\sum_{n=0}^{M-1}\frac{\gamma_{\mathrm{req}}^n e^{-\frac{\gamma_{\mathrm{req}}}{\sigma_x^2}}}{\sigma_x^{2n}\, n!\, \sigma_y^{2M}(M-1)!}$$

$$\times\int_0^\infty (1+y)^n e^{-\frac{\gamma_{\mathrm{req}}\, y}{\sigma_x^2}}\, y^{M-1}e^{-\frac{y}{\sigma_y^2}}\, dy. \quad (19)$$

Using Taylor series for expansion of $(1+y)^n$, the closed-form solution for integral in (19) is obtained, and thus, the outage probability can be written as

$$P_{\mathrm{out}}^{\mathrm{R}} = 1-e^{-\frac{\gamma_{\mathrm{req}}\lambda_0}{\sigma_{h_k}^2}}\sum_{n=0}^{M-1}\frac{\gamma_{\mathrm{req}}^n\lambda_0^n}{\sigma_{h_k}^{2n}\, n!\,(M-1)!}$$
$$\times\sum_{i=0}^{n}\binom{n}{i}\frac{(i+M-1)!\,(A_{\mathrm{R}}\sigma_{h_{-k}}^2)^i}{\left(1+\frac{A_{\mathrm{R}}\sigma_{h_{-k}}^2}{\sigma_{h_k}^2}\gamma_{\mathrm{req}}\right)^{i+M}\lambda_0^i}. \quad (20)$$

Now, we study the asymptotic behavior of receive outage probability. The outage probability in (20) can be expressed as $P_{\mathrm{out}}^{\mathrm{R}}(y,\Omega_k) = 1-e^{-y}g(y,\Omega_k)$ where $y=\frac{\gamma_{\mathrm{req}}\lambda_0}{\sigma_{h_k}^2}$, $\Omega_k = \frac{A_{\mathrm{R}}\sigma_{h_{-k}}^2}{\sigma_{h_k}^2}\gamma_k$, and

$$g(y,\Omega_k) = \sum_{n=0}^{M-1}\frac{y^n}{n!\,(M-1)!}\sum_{i=0}^{n}\binom{n}{i}\frac{(i+M-1)!\,\Omega_k^i}{(1+\Omega_k)^{i+M}\, y^i}. \quad (21)$$

Now, using Taylor series, we have $g(y,\Omega_k) = \sum_{m=0}^{M-1} U_m\, y^m$, where

$$U_m = \sum_{n=m}^{M-1}\frac{(M+n-m-1)!\,\Omega_k^{n-m}}{(n-m)!\, m!\,(M-1)!\,(1+\Omega_k)^{n-m+M}}, \quad (22)$$

for $m=0,\ldots,M-1$. Since $y=\frac{\gamma_{\mathrm{req}}\lambda_0}{\sigma_{h_k}^2} = \frac{\gamma_{\mathrm{req}}(1+A_{\mathrm{R}})}{\mathrm{SNR}_s\sigma_{h_k}^2}$, the outage probability of the system for high SNR scenarios can be obtained as

$$\lim_{\mathrm{SNR}_s\to\infty} P_{\mathrm{out}}^{\mathrm{R}} = \lim_{y\to 0} P_{\mathrm{out}}^{\mathrm{R}} = 1-U_0$$
$$= 1-\sum_{n=0}^{M-1}\frac{(M+n-1)!\,\Omega_k^n}{n!\,(M-1)!\,(1+\Omega_k)^{n+M}}. \quad (23)$$

Thus, unlike transmit I/Q imbalance case, the system with receive I/Q imbalance is interference-limited. Therefore, for a receive I/Q imbalanced beamforming MIMO OFDM system equipped with M transmit antennas and a single receive antenna, there is an error floor for the outage probability.

C. Outage Due to Both Transmit and Receive I/Q Imbalances

Next, we investigate the statistical properties of the system with transmit and receive I/Q imbalances, and a closed-form expression for outage probability is calculated at the k-th subcarrier. From (13), the outage probability at the k-th subcarrier can be represented as

$$P_{\mathrm{out}}^{\mathrm{TR}} \triangleq \Pr\left\{\mathrm{SINR}_k^{\mathrm{TR}} < \gamma_{\mathrm{req}}\right\}.$$

Lemma *2:* Consider a finite set of independent random variables $\mathcal{X}=\{X_1,\ldots,X_M\}$ and $\mathcal{Y}=\{Y_1,\ldots,Y_M\}$ with exponential distribution and mean σ_x^2 and σ_y^2, respectively. The CDF of

$$\mathrm{SINR} = \frac{\sum_{m=1}^{M} X_m}{1+A\sum_{m=1}^{M} X_m + \sum_{m=1}^{M} Y_m}$$

can be calculated as

$$\Pr\{\mathrm{SINR}<\gamma\} = 1-u(1-A\gamma)\Delta(\gamma) \quad (24)$$

where

$$\Delta(\gamma) = \sum_{i=0}^{M-1}\frac{\gamma^i e^{-\frac{\gamma}{\sigma_x^2(1-A\gamma)}}}{\sigma_x^{2i}\, i!\,(M-1)!\,(1-A\gamma)^i}$$
$$\times\sum_{j=0}^{i}\frac{\binom{i}{j}(j+M-1)!\,\sigma_y^{2j}}{\left(1+\frac{\sigma_y^2\gamma}{\sigma_x^2(1-A\gamma)}\right)^{j+M}}. \quad (25)$$

Proof: We define $Y=\sum_{m=1}^{M} Y_m$ which has an Erlang distribution M degrees of freedom with PDF $p_y(y) = \frac{y^{M-1}}{\sigma_y^{2M}(M-1)!}e^{-\frac{y}{\sigma_y^2}}$. Moreover, $X=\sum_{n=1}^{M} X_n$ has Erlang distribution with CDF of $\Pr\{X<x\} = 1-\sum_{n=0}^{M-1}\frac{x^n}{\sigma_x^{2n}\, n!}e^{-\frac{x}{\sigma_x^2}}$. By marginalizing over the random variable Y, when $\gamma<\frac{1}{A}$, the CDF of the $\mathrm{SINR}=\frac{X}{1+AX+Y}$ can be calculated as

$$\Pr\{\mathrm{SINR}<\gamma\} = \int_0^\infty \Pr\left\{X<\frac{\gamma(1+y)}{1-A\gamma}\right\} p_y(y)dy$$

$$= 1-\sum_{n=0}^{M-1}\int_0^\infty \frac{(\gamma+\gamma\, y)^n e^{-\frac{\gamma(1+y)}{\sigma_x^2(1-A\gamma)}}}{\sigma_x^{2n}\, n!\,(1-A\gamma)^n}\frac{y^{M-1}e^{-\frac{y}{\sigma_y^2}}}{\sigma_y^{2M}(M-1)!}dy$$

$$= 1-\sum_{n=0}^{M-1}\frac{\gamma^n e^{-\frac{\gamma}{\sigma_x^2(1-A\gamma)}}}{\sigma_x^{2n}\, n!\,\sigma_y^{2M}(M-1)!\,(1-A\gamma)^n}$$

$$\times\int_0^\infty (1+y)^n e^{-\frac{\gamma\, y}{\sigma_x^2(1-A\gamma)}}\, y^{M-1}e^{-\frac{y}{\sigma_y^2}}\, dy. \quad (26)$$

Using Taylor series for expansion of $(1+y)^n$, the closed-form solution for integral in (26) is obtained as in (24). It can be checked that $\Pr\{\mathrm{SINR}<\gamma\}=1$, when $\gamma\geq\frac{1}{A}$. This completes the proof. ■

From Lemma 2 and (13), when the condition $A_{\mathrm{T}}\gamma_{\mathrm{req}}<1$ or $R_k<\log_2\left(1+\frac{|\alpha_{\mathrm{T}}|^2}{|\beta_{\mathrm{T}}|^2}\right)$ is satisfied, the outage probability can be written as

$$P_{\mathrm{out}}^{\mathrm{TR}} \leq 1-\sum_{n=0}^{M-1}\frac{\gamma_{\mathrm{req}}^n\lambda_{\mathrm{TR}}^n e^{-\frac{\gamma_{\mathrm{req}}\lambda_{\mathrm{TR}}}{\sigma_{h_k}^2(1-A_{\mathrm{T}}\gamma_{\mathrm{req}})}}}{\sigma_{h_k}^{2n}\, n!\,(M-1)!\,(1-A_{\mathrm{T}}\gamma_{\mathrm{req}})^n}$$
$$\times\sum_{i=0}^{n}\binom{n}{i}\frac{(i+M-1)!\,(A_{\mathrm{R}}\sigma_{h_{-k}}^2)^i}{\left(1+\frac{A_{\mathrm{R}}\sigma_{h_{-k}}^2\gamma_{\mathrm{req}}}{\sigma_{h_k}^2(1-A_{\mathrm{T}}\gamma_k)}\right)^{i+M}\lambda_{\mathrm{TR}}^i}. \quad (27)$$

Proposition *2:* Consider a beamforming MIMO OFDM system equipped with M transmit antennas and a single receive antenna, in the existence of both transmit and receive I/Q imbalances. When the condition $R_k<\log_2\left(1+\frac{|\alpha_{\mathrm{T}}|^2}{|\beta_{\mathrm{T}}|^2}\right)$ is satisfied, there is an error floor for the outage probability for rate R_k, with the size of

$$\lim_{\mathrm{SNR}_s\to\infty} P_{\mathrm{out}}^{\mathrm{TR}} = 1-\sum_{n=0}^{M-1}\frac{(M+n-1)!\,\Phi_k^n}{n!\,(M-1)!\,(1+\Phi_k)^{n+M}}, \quad (28)$$

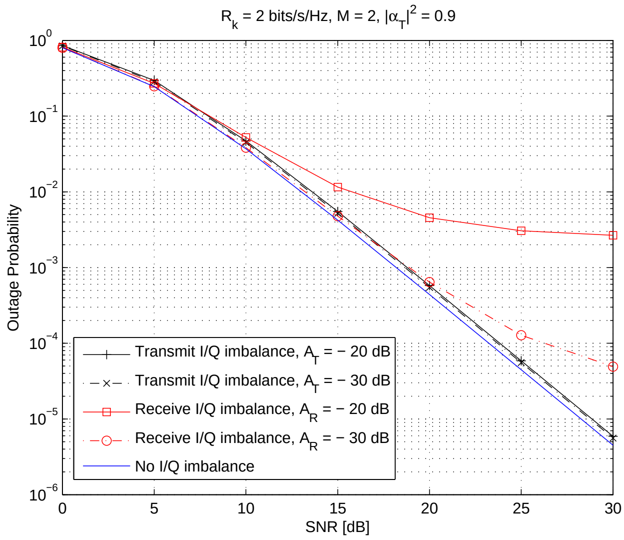

Fig. 1. The outage probability at the receiver P_{out} in a beamformed OFDM system with $M = 2$ transmit antennas, $R_k = 2$ bits/s/Hz, and $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$, for $k = 1, \ldots, K$, in presence of I/Q imbalances.

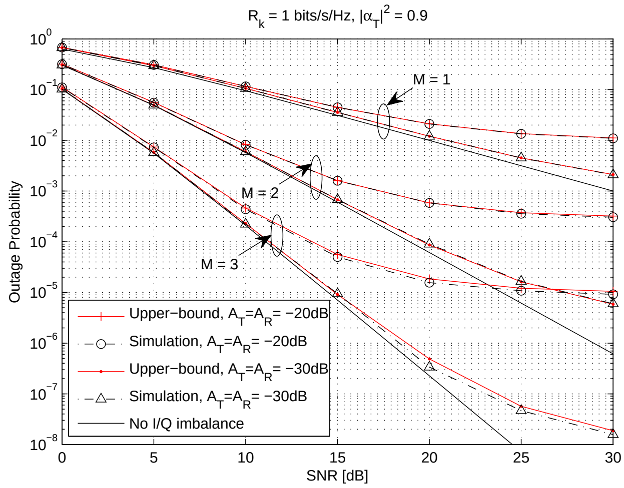

Fig. 2. Performance comparison of analytical and simulated results of an OFDM system with $M = 1, 2, 3$ transmit antenna(s), $R_k = 1$ bits/s/Hz, and $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$, for $k = 1, \ldots, K$, in presence of both transmit and receive I/Q imbalances.

where $\Phi_k = \frac{A_{\rm R}\sigma^2_{h_{-k}}}{\sigma^2_{h_k}(1+A_{\rm T}-2^{R_k}A_{\rm T})}\left(2^{R_k}-1\right)$.

Proof: The outage probability in (27) can be expressed as $P^{\rm R}_{\rm out}(z,\Phi) = 1 - e^{-z}g(z,\Phi)$ where $z = \frac{\gamma_{\rm req}\lambda_{\rm TR}}{\sigma^2_{h_k}(1-A_{\rm T}\gamma_{\rm req})}$. Now, using Taylor series, and from (21) and (22), we have $g(z,\Phi) = \sum_{m=0}^{M-1} V_m\, z^m$ where

$$V_m = \sum_{n=m}^{M-1} \frac{(M+n-m-1)!\,\Phi^{n-m}}{(n-m)!\,m!\,(M-1)!\,(1+\Phi)^{n-m+M}}, \quad (29)$$

for $m = 0, \ldots, M-1$. Since $z = \frac{\gamma_{\rm req}\lambda_{\rm TR}}{\sigma^2_{h_k}(1-A_{\rm T}\gamma_{\rm req})} = \frac{\gamma_{\rm req}(1+A_{\rm R})}{|\alpha_{\rm T}|^2\sigma^2_{h_k}(1-A_{\rm T}\gamma_{\rm req})\,{\rm SNR}_s}$, the outage probability of the system for high SNRs can be obtained as

$$\lim_{{\rm SNR}_s\to\infty} P^{\rm TR}_{\rm out} = \lim_{z\to 0} P^{\rm TR}_{\rm out} = 1 - V_0,$$

which yields to the result given in (28). ∎

IV. Numerical Analysis

In this section, numerical results are provided to demonstrate the effectiveness of our analytical results presented in previous sections. In all the evaluation scenarios, we have assumed that the channels are frequency selective fading and the frequency domain coefficients h_k and h_{-k} are independent Rayleigh distributed random variables with variance $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$.

Fig. 1 considers the outage probability experienced in the receiver as a function of the transmit SNR, i.e., $\frac{E_s}{N_0}$. We assume that the transmitter is equipped with $M = 2$ antennas and the transmission rate in each subcarrier R_k is fixed to 2 bits/s/Hz. The outage probability expression for a system exclusively consists of transmit or receive I/Q imbalance can be calculated by (16) and (20), respectively. For the case of transmit I/Q imbalance, we consider the transmit image-leakage-ratio $A_{\rm T}$ of -20 or -30 dB, and we set $|\alpha_{\rm T}|^2 = 0.9$. It can be observed from Fig. 1 that the transmit I/Q imbalance does not change the diversity order of the system, which confirms Theorem 1. It only reduce the coding gain by $1-|\alpha_{\rm T}|^2(A_{\rm T}(2^{R_k}-1))$, which is independent of the operating SNR. An intuitive explanation is that, in transmit I/Q imbalance, both the signal and its image experience the same fading realization. To see whether error floor will occur, let us set $N_0 = 0$. In this case, the SINR will be $1/A_T$. With the condition that $A_T < (2^{R_k}-1)^{-1}$, the outage probability will be zero (i.e. no error floor).For the case of receive I/Q imbalance, the receive image-leakage-ratio $A_{\rm R}$ of -20 or -30 dB is considered. In Fig. 1, it can be seen that the performance of the system significantly degraded due to the receive I/Q imbalance in high SNR scenarios. This confirms the results given in Theorem 2 that shows the system becomes interference-limited in high SNR scenarios. In the case of receive I/Q imbalance, the signal and image experience different fading channels; hence, the image can be greater than the signal itself even if $A_R < 1$. However, in low SNR scenario, unlike transmit I/Q imbalance case, the curves corresponded to the receive I/Q imbalances have the same performance as no I/Q imbalance case.

Fig. 2 confirms that the analytical outage probability expressions in Subsection III-C have similar performance as simulation results. In Fig. 2, we consider a network with $M = 1, 2, 3$ transmit antenna(s) and the transmission rate in each subcarrier R_k is fixed to 1 bits/s/Hz. The analytical results are based on (27), where it is assumed that both the transmitter and receiver are affected by the I/Q imbalances. It is shown that by increasing the leakage-image-ratio from -30 dB to -20 dB, the performance of the system is significantly impaired in high SNR conditions. Also there is a small reduction in coding gain in all SNR conditions due to the effect transmit I/Q imbalance. Fig. 2 confirms that the analytical results approximate the simulated results with a good precision. Note that the derived outage probability is based on the approximation in (13).

V. Conclusion

In this letter, we formulated the problem of finding the outage probability of a beamformed OFDM system over frequency selective fading channels with independent Rayleigh distributed coefficients, under different I/Q imbalance conditions. We first found various simple and exact closed-form expressions for a system impaired by either transmit or receive I/Q imbalances. Then, a tight approximation formula for the outage probability of the system with both the transmit and receive I/Q imbalances was derived. Furthermore, we have shown that the system with only transmit I/Q imbalance is not interference limited if the condition $A_T < (2^{R_k} - 1)^{-1}$ is satisfied, while receive I/Q imbalance leads to the error floor in high SNR scenarios.

VI. Acknowledgment

While this paper was in a review process, in May 2011, we (Behrouz Maham and Olav Tirkkonen) were shaken by the tragic loss of our friend and colleague Are Hjørungnes at the age of 40. Are was a dear colleague with a lot of energy, full of life and initiatives and passion for science. Our compassion and thoughts are with his family and friends for their loss.

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