Hindawi Publishing Corporation
EURASIP Journal on Advances in Signal Processing
Volume 2009, Article ID 612719, 13 pages
doi:10.1155/2009/612719

Research Article

Power Allocation Strategies for Distributed Space-Time Codes in Amplify-and-Forward Mode

Behrouz Maham[1,2] and Are Hjørungnes[1]

[1] *UNIK-University Graduate Center, University of Oslo, 2027 Kjeller, Norway*
[2] *Department of Electrical Engineering, Stanford University, Stanford, CA 94305, USA*

Correspondence should be addressed to Behrouz Maham, behrouz@unik.no

Received 22 February 2009; Revised 28 May 2009; Accepted 23 July 2009

Recommended by Jacques Palicot

We consider a wireless relay network with Rayleigh fading channels and apply distributed space-time coding (DSTC) in amplify-and-forward (AF) mode. It is assumed that the relays have statistical channel state information (CSI) of the local source-relay channels, while the destination has full instantaneous CSI of the channels. It turns out that, combined with the minimum SNR based power allocation in the relays, AF DSTC results in a new opportunistic relaying scheme, in which the best relay is selected to retransmit the source's signal. Furthermore, we have derived the optimum power allocation between two cooperative transmission phases by maximizing the average received SNR at the destination. Next, assuming M-PSK and M-QAM modulations, we analyze the performance of cooperative diversity wireless networks using AF opportunistic relaying. We also derive an approximate formula for the symbol error rate (SER) of AF DSTC. Assuming the use of full-diversity space-time codes, we derive two power allocation strategies minimizing the approximate SER expressions, for constrained transmit power. Our analytical results have been confirmed by simulation results, using full-rate, full-diversity distributed space-time codes.

1. Introduction

Space-time coding (STC) has received a lot of attention in the last years as a way to increase the data rate and/or reduce the transmitted power necessary to achieve a target bit error rate (BER) using multiple antenna transceivers. In ad hoc network applications or in distributed large-scale wireless networks, the nodes are often constrained in the complexity and size. This makes multiple-antenna systems impractical for certain network applications [1]. In an effort to overcome this limitation, cooperative diversity schemes have been introduced [1–4]. Cooperative diversity allows a collection of radios to relay signals for each other and effectively create a virtual antenna array for combating multipath fading in wireless channels. The attractive feature of these techniques is that each node is equipped with only *one* antenna, creating a virtual antenna array. This property makes them outstanding for deployment in cellular mobile devices as well as in ad hoc mobile networks, which have problem with exploiting multiple antenna due to the size limitation of the mobile terminals.

Among the most widely used cooperative strategies are amplify and forward (AF) [4, 5] and decode and forward (DF) [1, 2, 4]. The authors in [6] applied Hurwitz-Radon space-time codes in wireless relay networks and conjecture a diversity factor around $R/2$ for large R from their simulations, where R is the number of relays.

In [7], a cooperative strategy was proposed, which achieves a diversity factor of R in a R-relay wireless network, using the so-called distributed space time codes (DSTCs). In this strategy, a two-phase protocol is used. In phase one, the transmitter sends the information signal to the relays and in phase two, the relays send information to the receiver. The signal sent by every relay in the second phase is designed as a linear function of its received signal. It was shown in [7] that the relays can generate a linear space-time codeword at the receiver, as in a multiple antenna system, although they only cooperate distributively. This method does not require decoding at the relays and for high SNR it achieves the optimal diversity factor [7]. Although distributed space-time coding does not need instantaneous channel information at the relays, it requires full channel information at the receiver of both the channel from the transmitter to relays and the channel from relays to the receiver. Therefore, training symbols have to be sent from both the transmitter and relays. Distributed space-time coding was generalized to networks with multiple-antenna nodes in [8], and the design of practical DSTCs that lead to reliable communication in wireless relay networks has also been recently considered [9–11].

Power efficiency is a critical design consideration for wireless networks such as ad hoc and sensor networks, due to the limited transmission power of the nodes. To that end, choosing the appropriate relays to forward the source data as well as the transmit power levels of all the nodes become important design issues. Several power allocation strategies for relay networks were studied based on different cooperation strategies and network topologies in [12]. In [13], we proposed power allocation strategies for repetition-based cooperation that take both the statistical CSI and the residual energy information into account to prolong the network lifetime while meeting the BER QoS requirement of the destination. Distributed power allocation strategies for decode-and-forward cooperative systems are investigated in [14]. Power allocation in three-node models is discussed in [15, 16], while multihop relay networks are studied in [17–19]. Recent works also discuss relay selection algorithms for networks with multiple relays, which result in power efficient transmission strategies. Recently proposed practical relay selection strategies include preselect one relay [20], best-select relay [20], blind-selection algorithm [21], informed-selection algorithm [21], and cooperative relay selection [22]. In [23], an opportunistic relaying scheme is introduced. According to opportunistic relaying, a single relay among a set of R relay nodes is selected, depending on which relay provides the *best* end-to-end path between source and destination. Bletsas et al. [23] proposed two heuristic methods for selecting the best relay based on the end-to-end instantaneous wireless channel conditions. Performance and outage analysis of these heuristic relay selection schemes are studied in [24, 25]. In this paper, we propose a decision metric for opportunistic relaying based on maximizing the received instantaneous SNR at the destination in amplify-and-forward (AF) mode, when statistical CSI of the source-relay channel is available at the relay. Furthermore, similar to [7], knowledge of whole CSI is required for decoding at the destination. In this paper, we use a simple feedback from the destination toward the relays to select the best relay.

In [9, 10], a network with symmetric channels is assumed, in which all source-to-relay and relay-to-destination links have i.i.d. distributions. In [7], using the pairwise error probability (PEP) analysis in high SNR scenario, it is shown that uniform power allocation along relays is optimum. However, this assumption is hardly met in practice and the path lengths among nodes could vary. Therefore, power control among the relays is required for such a cooperation. In [10], a closed-form expression for the moment generating function (MGF) of AF space-time cooperation is derived as a function of Whittaker function. However, this function is not well behaved and cannot be used for finding an analytical solution for power allocation.

Our main contributions can be summarized as follows.

(i) We show that the DSTC based on [7] in which relays transmit the linear combinations of the scaled version of their received signals leads to a new opportunistic relaying, when maximum instantaneous SNR-based power allocation is used.

(ii) The optimum power allocation between two phases is derived by maximizing the average SNR at the destination.

(iii) We derive the average symbol error rate (SER) of AF opportunistic relaying system with M-PSK or M-QAM modulations over Rayleigh-fading channels. Furthermore, the probability density function (PDF) and moment generating function (MGF) of the received SNR at the destination are obtained.

(iv) We analyze the diversity order of AF opportunistic relaying based on the asymptotic behavior of average SER. Based on the proposed approximated SER expression, it is shown that the proposed scheme achieves the diversity order of R.

(v) The average SER of AF DSTC system for Rayleigh fading channels is derived, using two new methods based on MGF.

(vi) We propose two power allocation schemes for AF DSTC based on minimizing the target SER, given the knowledge of statistical CSI of source-relay links at the relays. An outstanding feature of the proposed schemes is that they are independent of the instantaneous channel variations, and thus, power control coefficients are varying slowly with time.

The rest of this paper is organized as follows. In Section 2, the system model is given. Power allocation schemes for AF DSTC based on minimizing the received SNR at the destination are presented in Section 3. In Section 4, the average SER of AF opportunistic relaying and AF DSTC with relays with partial statistical CSI is derived. Two power allocation schemes minimizing the SER are proposed in Section 5. In Section 6, the overall performance of the system is presented for different number of relays through simulations. Finally, Section 7 summarized the conclusions.

Throughout the paper, the following notation is applied. The superscripts ^{t} and ^{H} stand for transposition and conjugate transpose, respectively. $\mathbb{E}\{\cdot\}$ denotes the expectation operation. $\mathrm{Cov}(\mathbf{x}_T)$ is the covariance of the $T \times 1$ vector $\mathbf{x}_T$. All logarithms are the natural logarithm.

2. System Model

Consider the network in Figure 1 consisting of a source denoted s, one or more relays denoted Relay $r = 1, 2, \ldots, R$, and one destination denoted d. It is assumed that each node is equipped with a single antenna. We denote the source-to-rth relay and rth relay-to-destination links by f_r and g_r,

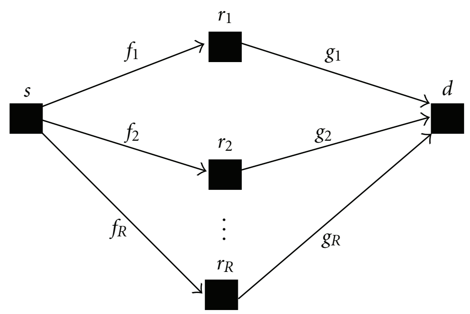

Figure 1: Wireless relay network consisting of a source s, a destination d, and R relays.

respectively. Suppose each link has a flat Rayleigh fading, and channels are independent of each others. Therefore, f_r and g_r are i.i.d. complex Gaussian random variables with zero-mean and variances $\sigma_{f_r}^2$ and $\sigma_{g_r}^2$, respectively. Similar to [7], our scheme requires two phases of transmission. During the first phase, the source node transmits a scaled version of the signal $\mathbf{s} = [s_1, \ldots, s_T]^t$, consisting of T symbols to *all* relays, where it is assumed that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = (1/T)\mathbf{I}_T$. Thus, from time 1 to T, the signals $\sqrt{P_1 T}s_1, \ldots, \sqrt{P_1 T}s_T$ are sent to all relays by the source. The average total transmitted energy in T intervals will be $P_1 T$. Assuming f_r is not varying during T successive intervals, the received $T \times 1$ signal at the rth relay can be written as

$$\mathbf{r}_r = \sqrt{P_1 T} f_r \mathbf{s} + \mathbf{v}_r, \tag{1}$$

where $\mathbf{v}_r$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with variance N_1. Using amplify and forward, each relay scales its received signal, that is,

$$\mathbf{y}_r = \rho_r \mathbf{r}_r, \tag{2}$$

where ρ_r is the scaling factor at Relay r. When there is no instantaneous CSI available at the relays, but statistical CSI is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is,

$$\rho_r^2 = \frac{P_{2,r}}{\sigma_{f_r}^2 P_1 + N_1}, \tag{3}$$

where $P_{2,r}$ is the average transmitted power at Relay r. The total power used in the whole network for one symbol transmission is therefore $P = P_1 + \sum_{r=1}^{R} P_{2,r}$.

DSTC, proposed in [7], uses the idea of linear dispersion space-time codes of multiple-antenna systems. In this system, the $T \times 1$ received signal at the destination can be written as

$$\mathbf{y} = \sum_{r=1}^{R} g_r \mathbf{A}_r \mathbf{y}_r + \mathbf{w}, \tag{4}$$

where $\mathbf{y}_r$ is given by (2), $\mathbf{w}$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with the component-wise variance of N_2, and the $T \times T$ dimensional matrix $\mathbf{A}_r$ is corresponding to the rth column of a proper $T \times T$ space-time code. The DSTCs designed in [9, 10] are such that $\mathbf{A}_r$, $r = 1, \ldots, R$, are unitary. Combining (1)–(4), the *total* noise vector $\mathbf{w}_T$ is given by

$$\mathbf{w}_T = \sum_{r=1}^{R} \mathbf{A}_r \sqrt{\frac{P_{2,r}}{\sigma_{f_r}^2 P_1 + N_1}} g_r \mathbf{v}_r + \mathbf{w}. \tag{5}$$

Since, g_i, $\mathbf{v}_i$, and $\mathbf{w}$ are independent complex Gaussian random variables, which are jointly independent, the conditional auto covariance matrix of $\mathbf{w}_T$ can be shown to be

$$\mathrm{Cov}\left(\mathbf{w}_T \mid \{f_r\}_{r=1}^{R}, \{g_r\}_{r=1}^{R}\right) = \left(\sum_{r=1}^{R} \frac{P_{2,r} |g_r|^2 N_1}{\sigma_{f_r}^2 P_1 + N_1} + N_2\right) \mathbf{I}_T, \tag{6}$$

where $\mathbf{I}_T$ is the $T \times T$ identity matrix. Thus, $\mathbf{w}_T$ is white.

3. Opportunistic Relaying through AF DSTC

In this section, we propose power allocation schemes for the AF distributed space-time codes introduced in [7], based on maximizing the received SNR at the destination d. First, the optimum power transmitted in the two phases, that is, P_1 and $P_2 = \sum_{r=1}^{R} P_{2,r}$, will be obtained by maximizing the average received SNR at the destination. Then, we will find the optimum distribution of transmitted powers among relays, that is, $P_{2,r}$, based on instantaneous SNR.

3.1. Power Control between Two Phases. In the following proposition, we derive the optimal value for the transmitted power in the two phases when backward and forward channels have different variances by maximizing the average SNR at the destination.

Proposition 1. *Assume α portion of the total power is transmitted in the first phase and the remaining power is transmitted by relays at the second phase, where $0 < \alpha < 1$, that is, $P_1 = \alpha P$ and $P_2 = (1-\alpha)P$, where P is the total transmitted power during two phases. Assuming $\sigma_{f_r}^2 = \sigma_f^2$ and $\sigma_{g_r}^2 = \sigma_g^2$, the optimum value of α by maximizing the average SNR at the destination is*

$$\alpha = \frac{N_1\sigma_g^2 P + N_1 N_2}{\left(N_2\sigma_f^2 - N_1\sigma_g^2\right)P}\left(\sqrt{1 + \frac{\left(N_2\sigma_f^2 - N_1\sigma_g^2\right)P}{N_1\sigma_g^2 P + N_1 N_2}} - 1\right). \tag{7}$$

Proof. The average SNR at the destination can be obtained by dividing the average received signal power by the variance of the noise at the destination (approximation of $\mathbb{E}\{\mathrm{SNR}\}$ using Jensen's inequality). Using (1)–(6), the average SNR can be written as

$$\mathrm{SNR} = \frac{\alpha(1-\alpha)P^2\sigma_f^2\sigma_g^2}{\alpha\left(N_2\sigma_f^2 - N_1\sigma_g^2\right)P + N_1\sigma_g^2 P + N_1 N_2}, \tag{8}$$

where we have assumed $\sigma_{f_r}^2 = \sigma_f^2$ and $\sigma_{g_r}^2 = \sigma_g^2$, for $r = 1, \ldots, R$, and thus, $P_{2,r} = P_2/R$. First, we consider the case in which $N_2\sigma_f^2 > N_1\sigma_g^2$. In this case, the optimum value of α

which maximizes (8), subject to the constraint $0 < \alpha < 1$, is obtained as

$$\alpha = \frac{\sqrt{1+\beta}-1}{\beta}, \tag{9}$$

where

$$\beta = \frac{\left(N_2\sigma_f^2 - N_1\sigma_g^2\right)P}{N_1\sigma_g^2 P + N_1 N_2}. \tag{10}$$

Similarly, when $N_2\sigma_f^2 < N_1\sigma_g^2$, the optimum value of α, which maximizes SNR in (8), subject to constraint $0 < \alpha < 1$, is also (9) and (10). Therefore, observing (9) and (10), the desired result in (7) is achieved. □

For the special case of $N_2\sigma_f^2 = N_1\sigma_g^2$, the optimum α is equal to 1/2, which is in compliance with the result obtained in [7], where assumed $N_1 = N_2$ and $\sigma_f^2 = \sigma_g^2$. In this case, we have

$$\begin{aligned}\alpha &= \lim_{\beta\to 0^+}\frac{1}{\beta}\left(\sqrt{1+\beta}-1\right)\\ &= \lim_{\beta\to 0^+}\frac{1}{\beta}\left(\frac{\beta}{2}+o(1)\right) = \frac{1}{2}.\end{aligned} \tag{11}$$

3.2. Power Control among Relays with Source-Relay link CSI at Relay. Now, we are going to find the optimum distribution of the transmitted powers among relays during the second phase, in a sense of maximizing the instantaneous SNR at the destination.

The conditional variance of the equivalent received noise is obtained in (6). Thus, using (1), (2), and (4), the instantaneous received SNR at the destination can be written as

$$\text{SNR}_{\text{ins}} = \frac{\sum_{r=1}^{R} P_1|f_r|^2|g_r|^2\left(P_{2,r}/\left(\sigma_{f_r}^2P_1+N_1\right)\right)}{\sum_{r=1}^{R}|g_r|^2\left(P_{2,r}/\left(\sigma_{f_r}^2P_1+N_1\right)\right)N_1+N_2}. \tag{12}$$

For notational simplicity, we represent SNR_{ins} in (12) in a matrix format as

$$\text{SNR}_{\text{ins}} = \frac{\mathbf{p}^t\mathbf{U}\mathbf{p}}{\mathbf{p}^t\mathbf{V}\mathbf{p}+N_2}, \tag{13}$$

where $\mathbf{p} = [\sqrt{P_{2,1}}, \sqrt{P_{2,2}}, \ldots, \sqrt{P_{2,R}}]^t$ and the positive definite diagonal matrices $\mathbf{U}$ and $\mathbf{V}$ are defined as

$$\begin{aligned}\mathbf{U} &= \text{diag}\left[\frac{P_1|f_1|^2|g_1|^2}{\sigma_{f_1}^2P_1+N_1}, \frac{P_1|f_2|^2|g_2|^2}{\sigma_{f_2}^2P_1+N_1}, \ldots, \frac{P_1|f_R|^2|g_R|^2}{\sigma_{f_R}^2P_1+N_1}\right],\\ \mathbf{V} &= \text{diag}\left[\frac{|g_1|^2N_1}{\sigma_{f_1}^2P_1+N_1}, \frac{|g_2|^2N_1}{\sigma_{f_2}^2P_1+N_1}, \ldots, \frac{|g_R|^2N_1}{\sigma_{f_R}^2P_1+N_1}\right].\end{aligned} \tag{14}$$

Then, the optimization problem is formulated as

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \text{SNR}_{\text{ins}}, \quad \text{subject to } \mathbf{p}^t\mathbf{p} = P_2, \tag{15}$$

where the $R\times 1$ vector $\mathbf{p}^*$ denotes the optimum values of power control coefficients. Moreover, since $\mathbf{p}^t\mathbf{p} = P_2 = (1-\alpha)P$, we can rewrite (13) as

$$\text{SNR}_{\text{ins}} = \frac{\mathbf{p}^t\mathbf{U}\mathbf{p}}{\mathbf{p}^t\mathbf{W}\mathbf{p}}, \tag{16}$$

where diagonal matrix $\mathbf{W}$ is defined as $\mathbf{W} = \mathbf{V} + (N_2/P_2)\mathbf{I}_R$. Since $\mathbf{W}$ is a positive semidefinite matrix, we define $\mathbf{q} \triangleq \mathbf{W}^{1/2}\mathbf{p}$, where $\mathbf{W} = (\mathbf{W}^{1/2})^t\mathbf{W}^{1/2}$. Then, (16) can be rewritten as

$$\text{SNR}_{\text{ins}} = \frac{\mathbf{q}^t\mathbf{Z}\mathbf{q}}{\mathbf{q}^t\mathbf{q}}, \tag{17}$$

where diagonal matrix $\mathbf{Z}$ is $\mathbf{Z} = \mathbf{U}\mathbf{W}^{-1}$. Now, using Rayleigh-Ritz theorem [26], we have

$$\frac{\mathbf{q}^t\mathbf{Z}\mathbf{q}}{\mathbf{q}^t\mathbf{q}} \le \lambda_{\max}, \tag{18}$$

where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{Z}$, which is corresponding to the largest diagonal element of $\mathbf{Z}$, that is,

$$\begin{aligned}\lambda_{\max} &= \max_{r\in\{1,\ldots,R\}}\lambda_r\\ &= \max_{r\in\{1,\ldots,R\}}\frac{P_1P_2|f_r|^2|g_r|^2}{P_2|g_r|^2N_1+N_2\left(\sigma_{f_r}^2P_1+N_1\right)}.\end{aligned} \tag{19}$$

The equality in (18) holds if $\mathbf{q}$ is proportional to the eigenvector of $\mathbf{Z}$ corresponding to $\lambda_{\max}$. Since $\mathbf{Z}$ is a diagonal matrix with real elements, the eigenvectors of $\mathbf{Z}$ are given by the orthonormal bases $\mathbf{e}_r$, defined $e_{r,l} = \delta_{r,l}$, $l = 1,\ldots,R$. Hence, the optimum $\mathbf{q}_{\max}$ can be chosen to be proportional to $\mathbf{e}_{r_{\max}}$. On the other hand, since $\mathbf{p} = \mathbf{W}^{-1/2}\mathbf{q}$, and $\mathbf{W}$ is a diagonal matrix, the optimum $\mathbf{p}^*$ is also proportional to $\mathbf{e}_{r_{\max}}$. Using the power constraint of the transmitted power in the second phase, that is, $\mathbf{p}^t\mathbf{p} = P_2$, we have $\mathbf{p}^* = \sqrt{P_2}\mathbf{e}_{r_{\max}}$. This means that for each realization of the network channels, the best relay should transmit all the available power P_2 and all other relays should stay silent. Hence, the optimum power allocation based on maximizing the instantaneous received SNR at the destination is to select the relay with the highest instantaneous value of $P_1P_2|f_r|^2|g_r|^2/(P_2|g_r|^2N_1+N_2(\sigma_{f_r}^2P_1+N_1))$.

3.3. Relay Selection Strategy. In the previous subsection, it is shown that the optimum power allocation of AF DSTC based on maximizing the instantaneous received SNR at the destination is to select the relay with the highest instantaneous value of $P_1P_2|f_r|^2|g_r|^2/(P_2|g_r|^2N_1+N_2(\sigma_{f_r}^2P_1+N_1))$. We assume the knowledge of magnitude of source-to-rth relay link to be available for the process of relay selection. The process of selecting the best relay could be done by the destination. This is feasible since the destination node should be aware of all channels for coherent decoding. Thus, the same channel information could be exploited for the purpose of relay selection. However, if we assume a distributed relay selection algorithm, in which relays independently decide to select the best relay among them, such as work done in [23], the knowledge of local channels f_r and g_r is required for the rth relay. The estimation of f_r and g_r can be done by transmitting a ready-to-send (RTS) packet and a clear-to-send (CTS) packet in MAC protocols.

4. Performance Analysis

4.1. Performance Analysis of the Selected Relaying Scheme

4.1.1. SER Expression. In the previous section, we have shown that the optimum transmitted power of AF DSTC system based on maximizing the instantaneous received SNR at the destination led to opportunistic relaying. In this section, we will derive the SER formulas of best relay selection strategy under the amplify-and-forward mode. For this reason, we should first derive the received SNR at the destination due to the rth relay, when other relays are silent, that is,

$$\gamma_r = \frac{P_1 P_2 |f_r|^2 |g_r|^2}{P_2 |g_r|^2 N_1 + N_2 \left(\sigma_{f_r}^2 P_1 + N_1\right)}. \tag{20}$$

In the following, we will derive the PDF of γ_r in (20), which is required for calculating the average SER.

Proposition 2. *For the γ_r in (20), the probability density function $p_r(\gamma_r)$ can be written as*

$$\begin{aligned} p_r(\gamma_r) &= 2A_r e^{-B_r\gamma_r} K_0\left(2\sqrt{A_r\gamma_r}\right) \\ &\quad + 2B_r\sqrt{A_r\gamma_r}e^{-B_r\gamma_r}K_1\left(2\sqrt{A_r\gamma_r}\right), \end{aligned} \tag{21}$$

where A_r and B_r are defined as

$$A_r = \frac{N_2\left(\sigma_{f_r}^2 P_1 + N_1\right)}{P_1 P_2 \sigma_{f_r}^2 \sigma_{g_r}^2}, \qquad B_r = \frac{N_1}{P_1 \sigma_{f_r}^2}, \tag{22}$$

and $K_\nu(x)$ is the modified Bessel function of the second kind of order ν [27].

Proof. The proof is given in Appendix A. □

Define $\gamma_{\max} \triangleq \max\{\gamma_1, \gamma_2, \ldots, \gamma_R\}$. The conditional SER of the best relay selection system under AF mode with R relays can be written as

$$P_e\left(R \mid \{f_r\}_{r=1}^R, \{g_r\}_{r=1}^R\right) = c\, Q\left(\sqrt{g\,\gamma_{\max}}\right), \tag{23}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-u^2/2}\,du$, and the parameters c and g are represented as

$$\begin{aligned} c_{\mathrm{QAM}} &= 4\frac{\sqrt{M}-1}{\sqrt{M}}, \qquad c_{\mathrm{PSK}} = 2, \\ g_{\mathrm{QAM}} &= \frac{3}{M-1}, \qquad g_{\mathrm{PSK}} = 2\sin^2\left(\frac{\pi}{M}\right). \end{aligned} \tag{24}$$

For calculating the average SER, we need to find the PDF of $\gamma_{\max}$. Thus, in the following proposition, we derive the PDF of the maximum of R random variables expressed in (20).

Proposition 3. *For the γ_r in (20), the probability density function of the maximum of the R random variables, γ_r, can be written as*

$$p_{\max}(\gamma) = \sum_{r=1}^R p_r(\gamma)\prod_{\substack{i=1 \\ i\neq r}}^R\left[1 - 2e^{-B_i\gamma}\sqrt{A_i\gamma}K_1\left(2\sqrt{A_i\gamma}\right)\right], \tag{25}$$

where $p_r(\gamma)$ is derived in (21).

Proof. The proof is given in Appendix B. □

Now, we are deriving the SER expression for the selection relaying scheme discussed in Section 3. Averaging over conditional SER in (23), we have the exact SER expression as

$$\begin{aligned} P_e(R) &= \int_0^\infty P_e\left(R \mid \{f_r\}_{r=1}^R, \{g_r\}_{r=1}^R\right) p_{\max}(\gamma)\, d\gamma \\ &= \int_0^\infty c\, Q\left(\sqrt{g\,\gamma}\right) p_{\max}(\gamma)\, d\gamma. \end{aligned} \tag{26}$$

Using the moment generating function approach, we can express $P_e(R)$ given in (26) as

$$\begin{aligned} P_e(R) &= \int_0^\infty \frac{c}{\pi}\int_0^{\pi/2} e^{-g\,\gamma/(2\sin^2\phi)} p_{\max}(\gamma)\, d\phi\, d\gamma \\ &= \frac{c}{\pi}\int_0^{\pi/2} M_{\max}\left(-\frac{g}{2\sin^2\phi}\right) d\phi, \end{aligned} \tag{27}$$

where $M_{\max}(-s) = \mathbb{E}_\gamma(e^{-s\gamma})$ is the moment generating function of $\gamma_{\max}$. In the following theorem, we state a closed-form expression for $M_{\max}(-s)$ in (27).

Theorem 1. *For the R independent random variables γ_r, which is stated in (20), the MGF of $\gamma_{\max} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_R\}$ is given by*

$$\begin{aligned} M_{\max}(-s) \approx{}& \left(\prod_{r=1}^R B_r\right)\sum_{r=1}^R \frac{(R-1)!}{(s+B_r)^R} e^{A_r/(2(s+B_r))} \\ &\times \Bigg\{ \frac{\sqrt{A_r(s+B_r)}}{B_r}(R-1)! \cdot W_{-R+(1/2),0} \\ &\quad \times \left(\frac{A_r}{s+B_r}\right) + R!\; W_{-R,(1/2)}\left(\frac{A_r}{s+B_r}\right)\Bigg\}, \end{aligned} \tag{28}$$

where $W_{a,b}(x)$ is Whittaker function of orders a and b (see, e.g., [27] and [28, equation 9.224]).

Proof. The proof is given in Appendix C. □

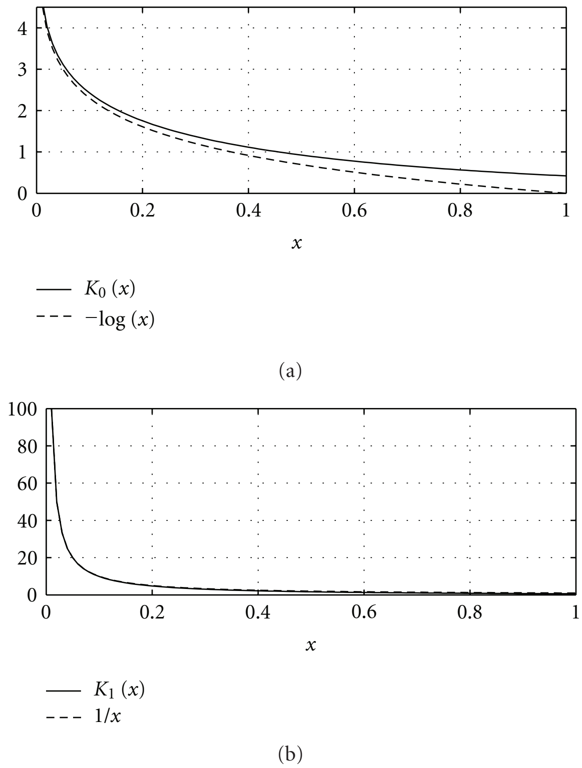

FIGURE 2: Diagrams of $K_0(x)$ and $\log(1/x)$ in (a) and $K_1(x)$ and $1/x$ in (b), which have the same asymptotic behavior when $x \to 0$.

4.1.2. Diversity Analysis. From [27, equation (9.6.8)], and [27, equation (9.6.9)], the following properties can be obtained

$$K_0(x) \approx -\log(x), \qquad K_1(x) \approx \frac{1}{x}. \tag{29}$$

Specially, for small values of x, which corresponds to the small value of A and B in (22), or equivalently, high SNR scenario, the approximations in (29) are more accurate. In Figure 2, we have shown that $K_0(x)$ and $\log(1/x)$, and also $K_1(x)$ and $1/x$ have the same asymptotic behavior when $x \to 0^+$. Therefore, we can approximate $p_r(\gamma)$ in (21) as

$$p_r(\gamma) \approx (B_r - A_r\log(4A_r))e^{-B_r\gamma} - A_r e^{-B_r\gamma}\log(\gamma), \tag{30}$$

and hence, $p_{\max}(\gamma)$ in (25) is approximated as

$$p_{\max}(\gamma) \approx \sum_{r=1}^{R}\Big[(B_r - A_r\log(4A_r))e^{-B_r\gamma} - A_r e^{-B_r\gamma}\log(\gamma)\Big] \times \prod_{\substack{i=1\\ i\neq r}}^{R}\Big(1 - e^{-B_r\gamma}\Big). \tag{31}$$

Using (31), we can approximate the moment generating function of $\gamma_{\max}$, that is, $M_{\max}(-s) = \mathbb{E}_\gamma(e^{-s\gamma})$, in high SNRs as

$$\begin{aligned} M_{\max}(-s) &= \int_0^\infty e^{-s\,\gamma} p_{\max}(\gamma)\,d\gamma \\ &\approx \sum_{r=1}^{R}\left(\prod_{\substack{i=1\\ i\neq r}}^{R} B_i\right)\int_0^\infty e^{-(s+B_r)\gamma} \\ &\quad \times \left[B_r - A_r\log(4A_r) - A_r\log(\gamma)\right]\gamma^{R-1}d\gamma, \end{aligned} \tag{32}$$

where we have approximated $(1 - e^{-B_r\gamma})$ with $B_r\gamma$, due to the high SNR assumption we made. Simplifying (32), we have

$$\begin{aligned} M_{\max}(-s) &\approx \sum_{r=1}^{R}\left(\prod_{\substack{i=1\\ i\neq r}}^{R} B_i\right)\left[B_r - A_r\log(4A_r)\right] \\ &\quad \times \int_0^\infty e^{-(s+B_r)\gamma}\gamma^{R-1}d\gamma \\ &\quad - \sum_{r=1}^{R} A_r\left(\prod_{\substack{i=1\\ i\neq r}}^{R} B_i\right)\int_0^\infty e^{-(s+B_r)\gamma}\log(\gamma)\gamma^{R-1}d\gamma, \end{aligned} \tag{33}$$

where the first integral can be calculated as $\int_0^\infty e^{-(s+B_r)\gamma}\gamma^{R-1}d\gamma = (R-1)!(s+B_r)^{-R}$. With the help of [28, equation (4.352)] , the second integral in (33) can be computed as

$$\begin{aligned} &\int_0^\infty e^{-(s+B_r)\gamma}\log(\gamma)\gamma^{R-1}d\gamma \\ &\quad = (R-1)!\ (s+B_r)^{-R}(\xi(R) - \log(s)), \end{aligned} \tag{34}$$

where $\xi(R) = 1 + 1/2 + 1/3 + \cdots + 1/(R-1) - \kappa$, and κ is the Euler's constant, that is, $\kappa \approx 0.5772156$. Therefore, the closed-form approximation for the MGF function of $\gamma_{\max}$ is given by

$$\begin{aligned} M_{\max}(-s) &\approx (R-1)!\sum_{r=1}^{R}\left(\prod_{\substack{i=1\\ i\neq r}}^{R} B_i\right)(s+B_r)^{-R} \\ &\quad \times \left[B_r - A_r\log(4A_r) + A_r(\log(s) - \xi(R))\right]. \end{aligned} \tag{35}$$

To have more insight into the MGF derived in (35), we represent A_r and B_r as functions of the transmit SNR, that is, $\mu = P/N_1$, assuming the destination and relays have the same value of noise, that is, $N_1 = N_2$. Thus, A_r and B_r in (22) can be represented in high SNRs as

$$A_r = \frac{1}{(1-\alpha)\mu\sigma_{g_r}^2}, \qquad B_r = \frac{1}{\alpha\mu\sigma_{f_r}^2}, \tag{36}$$

and then, $M_{\max}(-s)$ in (35) can be rewritten as

$$M_{\max}(-s) \approx \left(\prod_{i=1}^{R}\frac{1}{\sigma_{f_i}^2}\right)\sum_{r=1}^{R}\frac{(R-1)!}{[(s+B_r)\mu\alpha]^R} \times \left[1+\alpha\sigma_{f_r}^2\frac{\log\left(s\mu(1-\alpha)\sigma_{g_r}^2/4\right)-\xi(R)}{(1-\alpha)\sigma_{g_r}^2}\right]. \tag{37}$$

Now, we are using the moment generating function method to derive an approximate SER expression for the opportunistic relaying scheme discussed in Section 3. Using the moment generating function approach, we can express $P_e(R)$ given in (26) as

$$\begin{aligned}P_e(R) &= \int_0^{\infty}\frac{c}{\pi}\int_0^{\pi/2} e^{-\left(g\gamma/2\sin^2\phi\right)}\, p_{\max}(\gamma)\,d\phi\, d\gamma \\ &= \frac{c}{\pi}\int_0^{\pi/2} M_{\max}\left(-\frac{g}{2\sin^2\phi}\right)d\phi \\ &\approx \left(\prod_{i=1}^{R}\frac{1}{\sigma_{f_i}^2}\right)\frac{c2^R(R-1)!}{\pi(g\mu\alpha)^R}\sum_{r=1}^{R}\int_0^{\pi/2}\sin^{2R}\phi \\ &\quad\times\left[1+\alpha\sigma_{f_r}^2\frac{\log\left(g\mu(1-\alpha)\sigma_{g_r}^2/8\sin^2\phi\right)-\xi(R)}{(1-\alpha)\sigma_{g_r}^2}\right]d\phi,\end{aligned} \tag{38}$$

where by using (22), $g/2\sin^2\phi + B_r$ is accurately approximated with $g/2\sin^2\phi$ for all values of ϕ in high SNR conditions. For deriving the closed-form solution for the integral in (38), we decompose it into

$$P_e(R) \approx \Omega(\mu,R)\left[C_1(\mu,R)\int_0^{\pi/2}\sin^{2R}\phi\, d\phi - C_2(R) \times\int_0^{\pi/2}\sin^{2R}\phi\,\log(\sin\phi)\,d\phi\right], \tag{39}$$

where $\Omega(\mu,R)$, $C_1(\mu,R)$, and $C_2(R)$ are defined as

$$\Omega(\mu,R) = \frac{c2^R(R-1)!}{\pi(g\mu\alpha)^R}\prod_{i=1}^{R}\frac{1}{\sigma_{f_i}^2}, \tag{40}$$

$$C_1(\mu,R) = \sum_{r=1}^{R}\left[1+\alpha\sigma_{f_r}^2\frac{\log\left(g\mu(1-\alpha)\sigma_{g_r}^2/8\right)-\xi(R)}{(1-\alpha)\sigma_{g_r}^2}\right], \tag{41}$$

$$C_2(R) = \sum_{r=1}^{R}\frac{\alpha\sigma_{f_r}^2}{(1-\alpha)\sigma_{g_r}^2}. \tag{42}$$

Using [28, equation (4.387)] for solving the second integral in (39), the closed-form SER approximation is obtained as

$$P_e(R) \approx \frac{(2R)!}{((2^R R)!)^2}\frac{\pi}{2}\Omega(\mu,R) \times\left\{C_1(\mu,R) - C_2(R)\left(\sum_{k=1}^{R}\frac{(-1)^{k+1}}{k}-\log(2)\right)\right\}. \tag{43}$$

In the following theorem, we will study the achievable diversity gains in an opportunistic relaying network containing R relays, based on the SER expression.

Theorem 2. *The AF opportunistic relaying with the scaling factor presented in* (3)*, in which relays have no CSI, provides full diversity.*

Proof. The proof is given in Appendix D. □

4.2. SER Expression for AF DSTC. In this subsection, we derive approximate SER expressions for the AF space-time coded cooperation using moment generating function method.

The conditional SER of the protocol described in Section 2, with R relays, can be written as [29, equation (9.17)]

$$P_e\left(R \mid \{f_r\}_{r=1}^R\{g_r\}_{r=1}^R\right) = cQ\left(\sqrt{g\sum_{r=1}^{R}\mu_r\,|\,f_r g_r\,|^2}\right), \tag{44}$$

where by using (2)–(6), μ_r can be written as

$$\mu_r = \frac{P_1P_{2,r}/\left(\sigma_{f_r}^2P_1+N_1\right)}{\sum_{k=1}^{R}\left(P_{2,k}/\left(\sigma_{f_k}^2P_1+N_1\right)\right)\sigma_{g_k}^2N_1+N_2}. \tag{45}$$

It is important to note that in (45) we approximate the conditional variance of the noise vector $\mathbf{w}_T$ in (6) as its expected value. The received SNR at the receiver side is denoted

$$\gamma = \sum_{r=1}^{R}\gamma_r, \tag{46}$$

where

$$\gamma_r = \mu_r\,|\,f_r g_r\,|^2. \tag{47}$$

We can calculate the average SER as

$$\begin{aligned}P_e(R) &= \int_0^{\infty}P_e\left(R \mid \{\gamma_r\}_{r=1}^R\right)p(\gamma)\,d\gamma \\ &= \int_0^{\infty}c\,Q\left(\sqrt{g\,\gamma}\right)p(\gamma)\,d\gamma.\end{aligned} \tag{48}$$

Now, we are using the MGF method to calculate the SER expression in (48). We also exploit the property that the γ_r's are independent of each other, because of the inherit spatial separation of the relay nodes in the network. Hence, the average SER in (48) can be rewritten as

$$\begin{aligned} P_e(R) &= \int_{0;\,R-\text{fold}}^{\infty} \frac{c}{\pi}\int_0^{\pi/2}\prod_{r=1}^{R} e^{-(g\gamma_r/2\sin^2\phi)}\ d\phi \prod_{r=1}^{R}(p(\gamma_r)\,d\gamma_r) \\ &= \frac{c}{\pi}\int_0^{\pi/2}\int_{0;\,R-\text{fold}}^{\infty}\prod_{r=1}^{R}\left(e^{-(g\gamma_r/2\sin^2\phi)}p(\gamma_r)\ d\gamma_r\right)d\phi \\ &= \frac{c}{\pi}\int_0^{\pi/2}\prod_{r=1}^{R} M_r(-s)\,d\phi, \end{aligned} \tag{49}$$

where $M_r(-s)$ is the MGF of the random variable γ_r, and $s = g/2\sin^2\phi$.

It can be shown that for larger values of average SNR, $\overline{\gamma}$, the behavior of $\gamma/\overline{\gamma}$ becomes increasingly irrelevant because the Q term in (48) goes to zero so fast that almost throughout the whole integration range the integrand is almost zero. However, recalling that $Q(0) = 1/2$, regardless of the value of $\overline{\gamma}$, the behavior of $p(\gamma)$ around zero never loses importance. On the other hand, it is shown in [10, equation (19)] that the PDF of the random variables γ_r is proportional to the modified bessel function of second kind of zeroth order, that is,

$$p(\gamma_r) = \frac{2}{\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}K_0\left(2\sqrt{\frac{\gamma_r}{\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}}\right). \tag{50}$$

This PDF has a very large value around zero. Thus, the behavior of the integrand in (48) around zero becomes very crucial, and we can approximate $p(\gamma_r)$ in (50) with a logarithmic function, which is easier to handling. In Figure 2(a), we have shown that $K_0(x)$ and $\log(1/x)$ have the same asymptotic behavior when $x \rightarrow 0^+$, that is, $\lim_{x\rightarrow 0^+}K_0(x) \rightarrow -\log(x)$. Hence, we can approximate $M_r(-s)$ as

$$\begin{aligned} M_r(-s) &\approx \int_0^{\infty} e^{-s\,\gamma_r}\frac{-1}{\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\log\left(\frac{4\gamma_r}{\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\right)d\gamma_r \\ &= \frac{1}{s\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\left[\log\left(\frac{s\ \mu_r\sigma_{f_r}^2\sigma_{g_r}^2}{4}\right) - \kappa\right]. \end{aligned} \tag{51}$$

Furthermore, for the case of $R = 1$, the closed-form solution for the approximate SER is obtained as

$$\begin{aligned} P_e(R=1) &\approx \frac{c}{\pi}\int_0^{\pi/2} M(-s)\,d\phi \\ &= \frac{2c}{\pi g\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\int_0^{\pi/2}\sin^2\phi\left[\log\left(\frac{g\ \mu_r\sigma_{f_r}^2\sigma_{g_r}^2}{8\sin^2\phi}\right) - \kappa\right]d\phi \\ &= \frac{c}{2\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\left[\log\left(\frac{\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}{2}\right) - (\kappa+1)\right]. \end{aligned} \tag{52}$$

5. Power Control in AF DSTC without Instantaneous CSI at Relays

In this section, we propose two power allocation schemes for the AF distributed space-time codes introduced in [7]. We use the approximate value of the MGF, which was derived in Section 3, for the power control among relays. Furthermore, we present another closed-form solution for the MGF, as a function of the incomplete gamma function, which can be used for a more accurate power control strategy.

The MGF of the random variable γ, $M(-s)$, which is the integrand of the integral in (49), is given by the product of MGF of the random variables γ_r. Since $M_r(-s)$ is independent of the other μ_i, $i \neq r$, we can write

$$\frac{\partial M(-s)}{\partial\mu_r} = \frac{\partial M_r(-s)}{\partial\mu_r}\prod_{\substack{i=1\\ i\neq r}}^{R} M_i(-s), \tag{53}$$

which will be used in the next two subsections to find the power control coefficients.

5.1. Power Allocation Based on Exact MGF. The closed-form solution for MGF of random variable γ_r can be found using [28, equation (8.353)] as

$$\begin{aligned} M_r(-s) &= \frac{2}{s\ \mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\Gamma \\ &\quad\times\left(0, \frac{1}{s\ \mu_r\sigma_{f_r}^2\sigma_{g_r}^2}\right)e^{1/s\mu_r\sigma_{f_r}^2\sigma_{g_r}^2}, \end{aligned} \tag{54}$$

where $\Gamma(\alpha, x)$ is the incomplete gamma function of order α [27, equation (6.5)] . Moreover, from [28, (8.356)], we have

$$\frac{-d\ \Gamma(\alpha,x)}{dx} = x^{\alpha-1}e^{-x}. \tag{55}$$

Since the MGFs in (51) and (54) are functions of $x_r \triangleq \mu_r\sigma_{f_r}^2\sigma_{g_r}^2 s$, we can express (53) in terms of x_r. Hence, using (55), the partial derivative of $M_r(-s)$ with respect to x_r can be expressed as

$$\begin{aligned} \frac{\partial M_r(-s)}{\partial x_r} &= \frac{\partial}{\partial x_r}\left[\frac{2}{x_r}\Gamma\left(0,\frac{1}{x_r}\right)e^{1/x_r}\right] \\ &= \frac{1}{x_r^2}\left[1 - \Gamma\left(0,\frac{1}{x_r}\right)\left(1+\frac{1}{x_r}\right)\ e^{1/x_r}\right]. \end{aligned} \tag{56}$$

Furthermore, the power constraint in the the second phase, that is, $\sum_{r=1}^{R} P_{2,r} = P_1$, can be expressed as a function of x_r. Thus, using (45) and the definition of x_r, under the high SNR assumption, we have the following constraint:

$$\sum_{r=1}^{R}\frac{x_r}{\sigma_{g_r}^2 s} \leq \frac{P_1}{N_2}. \tag{57}$$

Given the objective function as an integrand of (49) and the power constraint in (57), the classical Karush-Kuhn-Tucker (KKT) conditions for optimality [30] can be shown as

$$\prod_{\substack{i=1\\ i\neq r}}^{R}\left[\frac{2}{x_i}\Gamma\left(0,\frac{1}{x_i}\right)e^{1/x_i}\right]\frac{1}{x_r^2}\times\left[1-\Gamma\left(0,\frac{1}{x_r}\right)\left(1+\frac{1}{x_r}\right)e^{1/x_r}\right]+\frac{\lambda}{\sigma_{g_r}^2 s}=0 \quad \text{for } r=1,\dots,R. \tag{58}$$

By solving (57) and (58), the optimum values of x_r, that is, x_r^*, $r = 1,\dots,R$ can be obtained. Now, we can have the following procedure to find the power control coefficients, $P_{2,r}$. First, the x_r^* coefficients can be solved by the above optimization problem. Then, recalling the relationship between x_r and μ_r, that is, $x_r = \mu_r\sigma_{f_r}^2\sigma_{g_r}^2 s$, and by taking average μ_r over different values of ϕ, since s is a function of $\sin^2\phi$, the optimum value of μ_r is obtained. However, for computational simplicity in the simulation results, we have assumed $s = 1$, which corresponds to $\phi = \pi/2$. Since the maximum amount of $M_r(-s)$ occurs in $s = 1$, this approximation achieves a good performance as will be confirmed in the simulation results. Finally, using (45), we can find the power control coefficients, $P_{2,r}$. If we assume that relays operate in the high SNR region, $P_{2,r}$ would be approximately proportional to μ_r.

5.2. Power Allocation Based on Approximate MGF. The power allocation proposed in Section 4.1 needs to solve the set of nonlinear equations presented in (58), which are function of incomplete gamma functions. Thus, we present an alternative scheme in this subsection. For gaining insight into the power allocation based on minimizing the SER, we are going to minimize the approximate MGF of the random variable γ, obtained in (51). Using (51) and (57), we can formulate the following problem:

$$\min_{\{x_1,x_2,\dots x_R\}}\prod_{r=1}^{R}\frac{1}{x_r}\left(\log\left(\frac{x_r}{4}\right)-\kappa\right), \quad \text{subject to } \sum_{r=1}^{R}\frac{x_r}{\sigma_{g_r}^2 s}\le\frac{P_1}{N_2},\quad x_r\ge 0,\ \text{for } r=1,\dots,R. \tag{59}$$

The objective function in (59), that is, $F(x_1,x_2,\dots,x_R) = \prod_{r=1}^{R}(1/x_r)(\log(x_r/4)-\kappa)$, is *not* a convex function in general. However, it can be shown that for $x_r > 4\ e^{1.5+\kappa}$, the Hessian of $F(x_1,x_2,\dots,x_R)$, is positive, which corresponds to high SNR conditions, this function is convex. Therefore, the problem stated in (59) is a convex problem for high SNR values and has a global optimum point. Now, we are going to derive a solution for a problem expressed in (59).

The Lagrangian of the problem stated in (59) is

$$L(x_1,x_2,\dots,x_R)=\prod_{r=1}^{R}\frac{\log(x_r)-\kappa'}{x_r}+\lambda\left(\sum_{r=1}^{R}\frac{x_r}{\sigma_{g_r}^2 s}-\frac{P_1}{N_2}\right), \tag{60}$$

where $\lambda > 0$ is the Lagrange multiplier, and $\kappa' = \log(4)+\kappa$. For nodes $r = 1,\dots,R$ with nonzero transmitter powers, the KKT conditions are

$$\left(-\frac{\log(x_r)}{x_r^2}+\frac{1+\kappa'}{x_r^2}\right)\prod_{\substack{i=1\\ i\neq r}}^{R}\frac{\log(x_i)-\kappa'}{x_i}+\frac{\lambda}{\sigma_{g_r}^2 s}=0. \tag{61}$$

Using (51) and some manipulations, one can rewrite (61) as

$$\left(\frac{1}{x_r}-\frac{1}{x_r(\log(x_r)-\kappa')}\right)M(s)=\frac{\lambda}{\sigma_{g_r}^2 s}. \tag{62}$$

Since the strong duality condition [30, equation (5.48)] holds for convex optimization problems, we have $\lambda(\sum_{r=1}^{R}(x_r/\sigma_{g_r}^2 s)-(P_1/N_2)) = 0$ for the optimum point. If we assume the Lagrange multiplier has a positive value, we have $\sum_{r=1}^{R}(x_r/\sigma_{g_r}^2 s) = P_1/N_2$. Therefore, by multiplying the two sides of (62) with x_r, and applying the summation over $r = 1,\dots,R$, we have

$$\left[R-\sum_{i=1}^{R}\frac{1}{\log(x_i)-\kappa'}\right]M(s)=\lambda\frac{P_1}{N_2}. \tag{63}$$

Dividing both sides of equalities in (62) and (63), we have

$$\frac{1}{x_r}\left(1-\frac{1}{\log(x_r)-\kappa'}\right)=\frac{N_2}{P_1\sigma_{g_r}^2 s}\left[R-\sum_{i=1}^{R}\frac{1}{\log(x_i)-\kappa'}\right] \tag{64}$$

for $r = 1,\dots,R$. The optimal values of x_r in the problem stated in (59) can be easily obtained with initializing some positive values for x_r, $r = 1,\dots,R$, and using (64) in an iterative manner. Then, we apply the same procedure stated in Section 4.1 to find the power control coefficients, $P_{2,r}$.

6. Simulation Results

In this section, the performance of the AF distributed space-time codes with power allocation is studied through simulations. We utilized distributed version of GABBA codes [10], as practical full-diversity distributed space-time codes, using BPSK modulation. We compare the transmit SNR (P/N_1) versus BER performance. We use the block fading model, in which channel coefficients changed randomly in time to isolate the benefits of spatial diversity. Assume that the relays and the destination have the same noise power, that is, $N_1 = N_2$.

In Figure 3, the BER performance of the AF DSTC is compared to the proposed AF opportunistic relaying derived

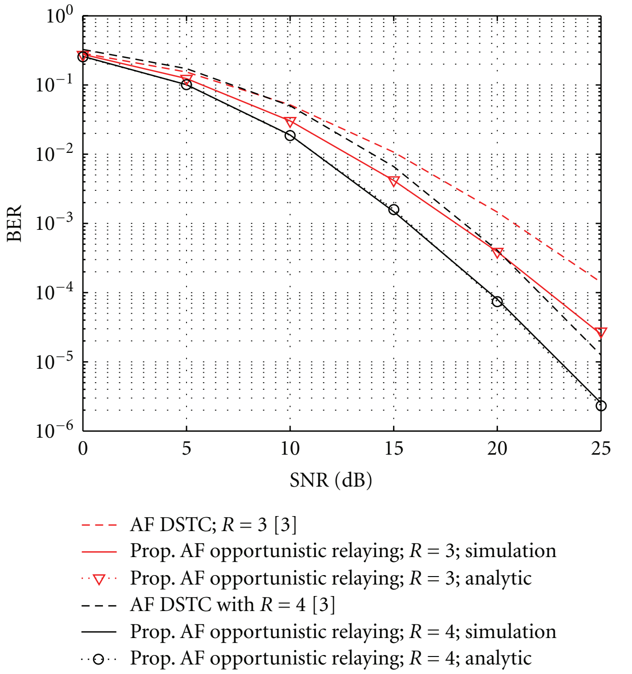

Figure 3: The average BER curves of relay networks employing DSTC and opportunistic relaying with partial statistical CSI at relays, BPSK signals and $\sigma_{f_i}^2 = \sigma_{g_i}^2 = 1$.

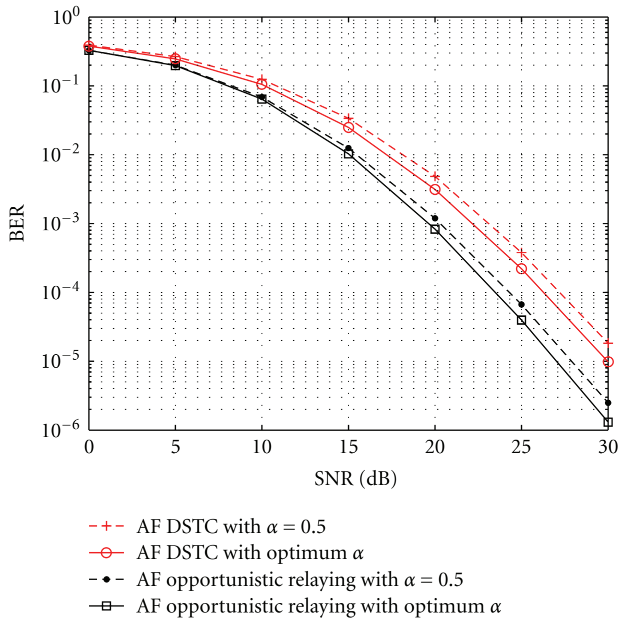

Figure 4: The average BER curves of relay networks employing DSTC and opportunistic relaying in AF mode, when equal power between two phases is compared with α in (7), and with BPSK signals, $\sigma_{f_i}^2 = 4\sigma_{g_i}^2 = 1$, and $R = 4$.

in Section 3, when the number of available relays is 3 and 4. For AF DSTC, equal power allocation is used among the relays. All links are supposed to have unit-variance Rayleigh flat fading. One can observe from Figure 3 that the AF opportunistic scheme gains around 2 and 3 dB in SNR at BER 10^{-3}, when 3 and 4 relays are used, respectively. Furthermore, Figure 3 confirms that the analytical results attained in Section 4 for finding SER for AF opportunistic relaying coincide with the simulation results. Since the curves corresponding to R relays are parallel to each other in the high SNR region, the AF opportunistic relaying has the same diversity gain as AF DSTC. In low SNR scenarios, due to the noise adding property of AF systems, even opportunistic relaying with $R = 3$ outperforms AF DSTC with $R = 4$.

Figure 4 compares the performance of the two AF schemes introduced in Section 3, when the proposed power allocation in two phases is employed. That is, we compare the equal power allocation in two phases [7] with the optimum value of α, which is derived in (7). The number of relays is supposed to be $R = 4$. Assuming $d_g = \sqrt{2}d_f = 2$, where d_f and d_g are source-to-relays and relays-to-destination distances, respectively, $\sigma_{f_i}^2 = 1/d_f^4 = 1$ and $\sigma_{g_i}^2 = 1/d_g^4 = 1/4$. This is due to the fact that path loss can be represented by $1/d^n$, where $2 < n < 5$, and we assume $n = 4$. Figure 4 demonstrates that by using the optimum value of α in (7), around 1 dB gain is achieved for both AF DSTC and AF opportunistic relaying schemes for BER of less than 10^{-3}. Therefore, the amount of performance gain obtainable using the optimal power allocation between two phases is negligible compared to the equal power allocation, that is, $\alpha = 1/2$.

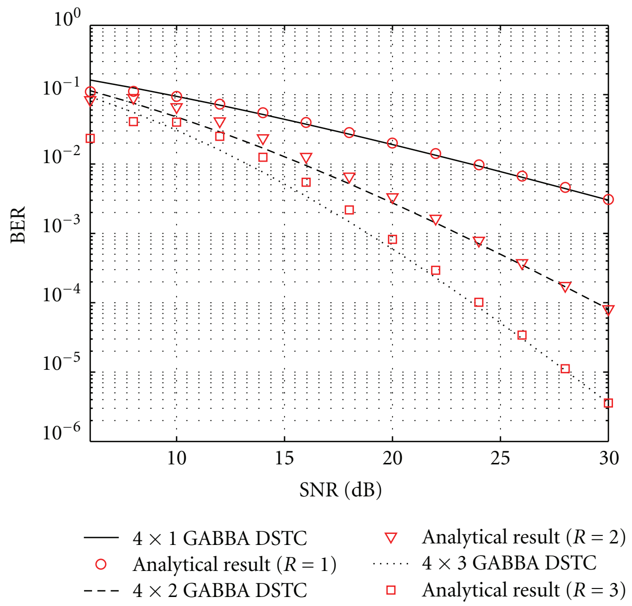

Figure 5: The average BER curves versus SNR of relay networks employing distributed space-time codes with BPSK signals.

In Figure 5, we compare the approximate BER formula based on MGF given in (51) with the full-rate, full-diversity distributed GABBA space-time codes. For GABBA codes, we employed 4×4 GABBA mother codes, that is, $T = 4$ [10]. Assume all the links have unit-variance Rayleigh flat fading. Figure 5 confirms that the analytical results attained

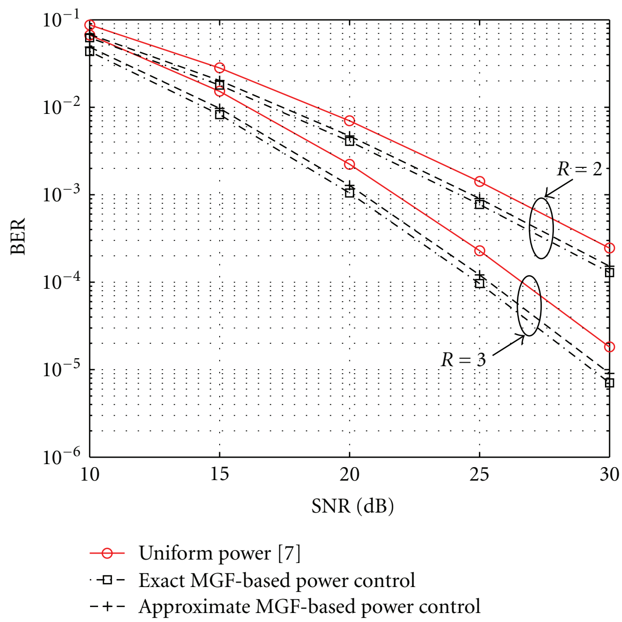

FIGURE 6: Performance comparison of AF DSTC with different power allocation strategies in a network with two and three relays and using BPSK signals.

in Section 3 for finding the BER approximate well the performance of the practical full-diversity distributed space-time codes for high SNR values.

Figure 6 presents the BER performance of the AF distributed space-time codes using different power allocation schemes. For transmission power among nodes, we employed the two power control schemes introduced in Section 4, and also uniform power transmission among relays, that is, $P_1 = P/2$ and $P_{2,r} = P/2R$ [7]. Since the proposed power allocation strategies are designed for high SNR scenarios, we study the system performance in the high SNR regime. Furthermore, since we supposed that the relays are operating in low noise conditions, here, we assume $N_2 = 2N_1$. Slow Rayleigh flat fading channels are considered, with variance of $\sigma^2_{f_r}(r) = \sigma^2_{g_r}(r) = \frac{1}{2^{r-1}}, r = 1, 2, \ldots, R$. For the power control scheme expressed in Section 4.1 (based on the exact MGF), we have used MATLAB optimization toolbox command "fmincon" designed to find the minimum of the given constrained nonlinear multivariable function. Figure 6 demonstrates that using the power control schemes of Section 5, about 1 and 2 dB gain will be obtained for $R = 2$ and $R = 3$ cases, respectively, comparing to uniform power allocation. The power control strategy given in Section 5.1 (exact MGF-based power control) has a slightly better performance than the power control strategy presented in Section 5.2 (Approximate MGF-based power control), at the expense of higher computational complexity.

7. Conclusion

In this paper, we have shown that using maximum instantaneous SNR power allocation at the relays, subject to the fixed transmit power during the second phase, distributed space-time codes under amplify and forward led to opportunistic relaying. Therefore, the whole transmission power during the second phase is transmitted by the relay with the best channel conditions. We analyzed the SER performance of the AF opportunistic relaying system with M-PSK and M-QAM signals. Simulations are in accordance with the analytic expressions. We also derived approximate BER formulas of AF DSTC using the moment generating function method, when M-PSK and M-QAM modulations are employed. Simulation results confirmed that the theoretical expressions have a similar performance to the Monte Carlo simulations at high SNR values. Furthermore, we proposed two power allocation methods based on minimizing the BER, which are independent of the knowledge of instantaneous CSI. Simulations showed that up to 2 dB is achieved in the high SNR region compared to an equal power transmission, when using three relays.

Appendices

A. Proof of Proposition 2

Suppose $X = |f_r|^2$ and $Y = |g_r|^2$, where X and Y have exponential distribution with mean of $\overline{X} = \sigma^2_{f_r}$ and $\overline{Y} = \sigma^2_{g_r}$, respectively. Therefore, the cumulative density function (CDF) of $Z = XY/(aY + b)$, where $a = N_1$ and $b = N_2(\sigma^2_{f_r}P_1 + N_1)/P_2$, can be presented to be

$$\begin{aligned}\Pr\{Z < z\} &= \Pr\{XY/(aY + b) < z\} \\ &= \int_0^\infty \Pr\left\{X < \frac{z(ay + b)}{y}\right\} p_Y(y)\,dy \\ &= \frac{1}{\overline{Y}}\int_0^\infty \left(1 - e^{-z(ay+b)/\overline{X}y}\right) e^{-y/\overline{Y}}\,dy \\ &= 1 - \frac{1}{\overline{X}}\int_0^\infty e^{-az/\overline{X}} e^{-\left(bz/\overline{X}y + y/\overline{Y}\right)}\,dy \\ &= 1 - 2\, e^{-az/\overline{X}}\sqrt{\frac{bz}{\overline{X}\overline{Y}}}K_1\left(2\sqrt{\frac{bz}{\overline{X}\overline{Y}}}\right),\end{aligned} \tag{A.1}$$

where we have used [28, equation (3.324)] for the last equality. The PDF of Z can be written as

$$p_Z(z) = \frac{d}{dz}\Pr\{Z < z\} = f_1(z) + f_2(z), \tag{A.2}$$

where $f_1(z)$ and $f_2(z)$ are defined as

$$f_1(z) = \frac{2b}{\overline{X}\overline{Y}} e^{-az/\overline{X}} K_0\left(2\sqrt{\frac{bz}{\overline{X}\overline{Y}}}\right), \tag{A.3}$$

$$f_2(z) = \frac{2a}{\overline{X}}\sqrt{\frac{bz}{\overline{X}\overline{Y}}} e^{-az/\overline{X}} K_1\left(2\sqrt{\frac{bz}{\overline{X}\overline{Y}}}\right), \tag{A.4}$$

where for the derivative of $(d/dz)\Pr\{Z < z\}$ we have used the following equality [27]

$$x\frac{d}{dx}K_\nu(x) = -xK_{\nu-1}(x) - \nu K_\nu(x). \tag{A.5}$$

Now, using (A.2)–(A.4), and the fact that the PDF of the random variable $\gamma_r = P_1 Z$ is $(1/P_1)p_Z(\gamma_r/P_1)$, we obtain the result in (21).

B. Proof of Proposition 3

For deriving the PDF of $\gamma_{\max}$ we should first find its CDF, which can be written as

$$\begin{aligned}\Pr\{\gamma_{\max} < \gamma\} &= \Pr\{\gamma_1 \le \gamma, \gamma_2 \le \gamma, \ldots, \gamma_R \le \gamma\} \\ &= \prod_{r=1}^{R}\Pr\{\gamma_r \le \gamma\}.\end{aligned} \tag{B.1}$$

The second equality comes from the fact that we assumed that all channel coefficients are independent of each others. Then the PDF of $\gamma_{\max}$ can be written as

$$\begin{aligned}p_{\max}(\gamma) &= \frac{d}{d\gamma}\Pr\{\gamma_{\max} < \gamma\} \\ &= \sum_{r=1}^{R}p_r(\gamma)\prod_{\substack{i=1\\i\neq r}}^{R}\Pr\{\gamma_i \le \gamma\}.\end{aligned} \tag{B.2}$$

Replacing $p_r(\gamma)$ and $\Pr\{\gamma_i \le \gamma\}$ from (21) and (A.1), respectively, in (B.2), the result given in (25) is obtained.

C. Proof of Theorem 1

Considering $p_{\max}(\gamma)$ stated in (25), we can express $M_{\max}(-s)$ as

$$\begin{aligned}M_{\max}(-s) &= \int_0^\infty e^{-s\gamma}p_{\max}(\gamma)\,d\gamma \\ &\approx \sum_{r=1}^{R}\int_0^\infty e^{-s\gamma}p_r(\gamma)\prod_{\substack{i=1\\i\neq r}}^{R}\left(1-e^{-B_i\gamma}\right)d\gamma \\ &\approx 2\sum_{r=1}^{R}\int_0^\infty e^{-(s+B_r)\gamma} \\ &\quad\times\left[A_rK_0\left(2\sqrt{A_r\gamma}\right)+B_r\sqrt{A_r\gamma}K_1\left(2\sqrt{A_r\gamma}\right)\right] \\ &\quad\times\left(\prod_{\substack{i=1\\i\neq r}}^{R}B_i\right)\gamma^{R-1}d\gamma,\end{aligned} \tag{C.1}$$

where in the second equality we have approximated $K_1(x) \approx 1/x$ (see, e.g., [27, equation (9.6.8)]), and in the third equality $(1-e^{-B_i\gamma})$ is approximated by $B_i\gamma$. These approximations are accurate for all values of B_i, since the fact that e^{-x}, $K_0(x)$, and $K_1(x)$ are decreasing functions of x in the integrand in (C.1), and thus, the value of $B_i\gamma$ around $\gamma = 0$ is critical. Simplifying (C.1), we get

$$\begin{aligned}M_{\max}(-s) &\approx 2\sum_{r=1}^{R}A_r\left(\prod_{\substack{i=1\\i\neq r}}^{R}B_i\right) \\ &\quad\times\int_0^\infty e^{-(s+B_r)\,\gamma}K_0\left(2\sqrt{A_r\gamma}\right)\gamma^{R-1}d\gamma \\ &\quad+2\sum_{r=1}^{R}\sqrt{A_r}\left(\prod_{r=1}^{R}B_r\right) \\ &\quad\times\int_0^\infty e^{-(s+B_r)\gamma}K_1\left(2\sqrt{A_r\gamma}\right)\gamma^{R-(1/2)}d\gamma.\end{aligned} \tag{C.2}$$

The integrals in (C.2) denoted I_1 and I_2, respectively, can be evaluated with the help of [28, equation (6.631)], which with some extra manipulations leads to

$$I_1 = \frac{\Gamma^2(R)(s+B_r)^{-R+(1/2)}}{2\sqrt{A_r}}e^{A_r/2(s+B_r)}W_{-R+(1/2),0}\left(\frac{A_r}{s+B_r}\right), \tag{C.3}$$

$$I_2 = \frac{\Gamma(R+1)\Gamma(R)(s+B_r)^{-R}}{2\sqrt{A_r}}e^{A_r/2(s+B_r)}W_{-R,1/2}\left(\frac{A_r}{s+B_r}\right), \tag{C.4}$$

where $\Gamma(n)$ is the gamma function of order n. Combining (C.2), (C.3), and (C.4), the desired result given in (28) is achieved.

D. Proof of Theorem 2

From (41)–(43), and by using a tractable definition of the diversity gain in [31, equation (1.19)], we have

$$\begin{aligned}G_d &= -\lim_{\mu\to\infty}\frac{\log(P_e(R))}{\log(\mu)} \\ &= -\lim_{\mu\to\infty}\frac{\log(\Omega(\mu,R))+\log(C_1(\mu,R))}{\log(\mu)} \\ &= -\lim_{\mu\to\infty}\frac{\log(\mu^{-R})}{\log(\mu)} - \lim_{\mu\to\infty}\frac{\log(\log(\mu))}{\log(\mu)} = R,\end{aligned} \tag{D.1}$$

where in the second, third, and fourth equations, we have used the l'Hôpital's rule. Hence, it is proven that AF opportunistic relaying scheme derived in Section 3 provides full diversity of order R in a network consisting of R relays.

Acknowledgment

This work was supported by the Research Council of Norway through the project 176773/S10 entitled "Optimized Heterogeneous Multiuser MIMO Networks – OptiMO".

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—part I: system description," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927–1938, 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—part II: implementation aspects and performance analysis," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1939–1948, 2003.

[3] J. N. Laneman and G. W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," in *Proceedings of IEEE Wireless Communications and Networking Conference*, pp. 7–12, Chicago, Ill, USA, September 2000.

[4] J. N. Laneman and G. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM '02)*, vol. 1, pp. 77–81, Taipei, Taiwan, November 2002.

[5] R. U. Nabar, H. Bölcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1099–1109, 2004.

[6] Y. Hua, Y. Mei, and Y. Chang, "Wireless antennas-making wireless communications perform like wireline communications," in *Proceedings of IEEE AP-S Topical Conference on Wireless Communication Technology*, Honolulu, Hawaii, USA, October 2003.

[7] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Transactions on Wireless Communications*, vol. 5, no. 12, pp. 3524–3536, 2006.

[8] Y. Jing and B. Hassibi, "Cooperative diversity in wireless relay networks with multiple-antenna nodes," in *Proceedings of International Symposium on Information Theory (ISIT '05)*, pp. 815–819, Adelaide, Australia, September 2005.

[9] Y. Jing and H. Jafarkhani, "Using orthogonal and quasi-orthogonal designs in wireless relay networks," *IEEE Transactions on Information Theory*, vol. 53, no. 11, pp. 4106–4118, 2007.

[10] B. Maham, A. Hjørungnes, and G. Abreu, "Distributed GABBA space-time codes in amplify-and-forward relay networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 4, pp. 2036–2045, 2009.

[11] G. S. Rajan and B. S. Rajan, "Distributed space-time codes for cooperative networks with partial CSI," in *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC '07)*, pp. 903–907, Hong Kong, March 2007.

[12] Y.-W. Hong, W.-J. Huang, F.-H. Chiu, and C.-C. J. Kuo, "Cooperative communications in resource-constrained wireless networks," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 47–57, 2007.

[13] B. Maham and A. Hjørungnes, "Minimum power allocation in SER constrained amplify-and-forward cooperation," in *Proceedings of the 67th IEEE Vehicular Technology Conference (VTC '08)*, pp. 2431–2435, Singapore, May 2008.

[14] M. Chen, S. Serbetli, and A. Yener, "Distributed power allocation strategies for parallel relay networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 2, pp. 552–561, 2008.

[15] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2020–2040, 2005.

[16] D. R. Brown, "Energy conserving routing in wireless adhoc networks," in *Proceedings of the 38th Asilomar Conference on Signals, Systems and Computers (ACSSC '04)*, Monterey, Calif, USA, November 2004.

[17] A. Reznik, S. R. Kulkarni, and S. Verdú, "Degraded Gaussian multirelay channel: capacity and optimal power allocation," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3037–3046, 2004.

[18] M. O. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, pp. 1999–2004, 2004.

[19] M. Dohler, A. Gkelias, and H. Aghvami, "Resource allocation for FDMA-based regenerative multihop links," *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, pp. 1989–1993, 2004.

[20] J. Luo, R. S. Blum, L. J. Cimini, L. J. Greenstein, and A. M. Haimovich, "Link-failure probabilities for practical cooperative Relay networks," in *Proceedings of the 61th IEEE Vehicular Technology Conference (VTC '05)*, pp. 1489–1493, Stockholm, Sweden, May 2005.

[21] Z. Lin and E. Erkip, "Relay search algorithms for coded cooperative systems," in *Proceedings of IEEE Global Telecommunications Conference (GLOBECOM '05)*, vol. 3, pp. 1314–1319, St. Louis, Mo, USA, November-December 2005.

[22] H. Zheng, Y. Zhu, C. Shen, and X. Wang, "On the effectiveness of cooperative diversity in ad hoc networks: a MAC layer study," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '05)*, vol. 3, pp. 509–512, Philadelphia, Pa, USA, March 2005.

[23] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 659–672, 2006.

[24] A. Bletsas, H. Shin, and M. Z. Win, "Outage optimality of opportunistic amplify-and-forward relaying," *IEEE Communications Letters*, vol. 11, no. 3, pp. 261–263, 2007.

[25] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Communications Letters*, vol. 10, no. 11, pp. 757–759, 2006.

[26] R. Horn and C. Johnson, *Matrix Analysis*, Cambridge Academic Press, Cambridge, UK, 1985.

[27] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, NY, USA, 1972.

[28] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic, San Diego, Calif, USA, 1996.

[29] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*, John Wiley & Sons, New York, NY, USA, 2000.

[30] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2004.

[31] H. Jafarkhani, *Space-Time Coding Theory and Practice*, Cambridge Academic Press, Cambridge, UK, 2005.