Symbol error rate of amplify-and-forward distributed space–time codes over Nakagami-*m* fading channel

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The performance of distributed space–time codes using amplify-and-forward relaying over independent, non-identical, Nakagami-*m* fading channels is analysed. The symbol error rate is derived using the moment generating function (MGF). The probability density function and MGF of the total SNR are first derived. Then, the MGF is used to determine the error rate.

Introduction: In [1], a co-operative strategy was proposed, which achieves a diversity factor of R in R-relay wireless networks, using the so-called distributed space–time codes (DSTC). In this strategy, a two-phase protocol is used. In phase one, the transmitter sends the information signal to the relays, and in phase two, the relays send information to the receiver. The signal sent by every relay in the second phase is designed as a linear function of its received signals and their complex conjugate. The design of practical DSTC has been recently considered [2]. Here, we derive the approximate average symbol error rate (SER) for amplify-and-forward (AF) DSTC with M-PSK modulation over independent, non-identical, Nakagami-*m* fading channels. The SER expression is valid for any full-diversity, full-rate space–time block codes, such as codes given in [2].

System model: Consider a source node and one destination node communicating over a channel with a flat Nakagami-*m* fading coefficient h. A number of co-operating nodes (r_i, $i = 1, 2, \ldots, R$) relay the signal to provide the destination with multiple copies of the original signal. The channel coefficients between the source and r_i (f_i) and between r_i and the destination (g_i) are also flat Nakagami-*m* fading coefficients. In addition, h, f_i, and g_i are mutually-independent and non-identical. We also assume, without loss of generality, that all additive white Gaussian noise terms have zero mean and equal variance N_0. Relays simply amplify and forward the the received signal with the scaling factor $\rho_i = \sqrt{\frac{\varepsilon_i}{\mathbb{E}[|f_i|^2]\varepsilon_s + N_0}}$ which is independent of the knowledge of the instantaneous CSI $\mathbb{E}[\cdot]$ denotes the expectation operation, and ε_s and ε_i are the source and ith relay transmitted power, respectively. Then, the relays simultaneously transmitting the scaled vector using DSTC. Similar to [1], we consider equal transmitting power, i.e. $\varepsilon_i = \frac{\varepsilon_s}{R}$. Assuming MRC at the destination, the total SNR at the destination can be approximated as $\gamma_d \simeq \gamma_h + \sum_{i=1}^{R}\gamma_i$, where $\gamma_0 = |h|^2\varepsilon_s/N_0$ is the instantaneous SNR between the source and ith relay, and γ_i can be presented to be

$$\gamma_i = \frac{\varepsilon_s^2}{R(\mathbb{E}[|f_i|^2]\varepsilon_s + N_0)}\frac{|f_i|^2|g_i|^2}{\frac{1}{R}\sum_{i=1}^{R}\varepsilon_s N_0\mathbb{E}[|g_i|^2]/(\mathbb{E}[|f_i|^2]\varepsilon_s + N_0) + N_0} \quad (1)$$

SER formula for AF DSTC: With the help of [3, eqn. (3.324)], the cumulative density function (CDF) of γ_i in (1) can be presented to be

$$\Pr\{\gamma_i < \gamma\}\int_0^\infty\left(1 - \frac{\Gamma(m_{f_i}, m_{f_i}\gamma\mathbb{E}[|g_i|^2]/y\bar{\gamma}_i)}{\Gamma(m_{f_i})}\right)p_Y(y)dy \quad (2)$$

where $\Gamma(\alpha,\ x)$ is the incomplete gamma function of order α [3, eqn. (8.350)], $\bar{\gamma}_i = \mathbb{E}[\gamma_i]$, $p_Y(y) = \frac{m_{g_i}^{m_{g_i}} y^{m_{g_i}-1}}{\mathbb{E}[|g_i|^2]^{m_{g_i}}\Gamma(m_{g_i})} e^{-\frac{m_{g_i}y}{\mathbb{E}[|g_i|^2]}}$, and m_{fi} and m_{gi} are the Nakagami-*m* fading parameters of f_i and g_i, respectively. Then, using (2) and $\frac{-d\Gamma(\alpha,x)}{dx} = x^{\alpha-1}e^{-x}$ [3, eqn. (8.356)], the PDF of γ_i can be written as

$$\begin{aligned} p_{\gamma_i}(\gamma) = \frac{d}{d_\gamma}\Pr\{\gamma_i < \gamma\} &= \frac{1}{\gamma}\left(\frac{\gamma\mathbb{E}[|f_i|^2]\mathbb{E}[|g_i|^2]}{\bar{\gamma}_i}\right)^{m_{f_i}+m_{g_i}/2} \\ &\times \frac{m_{f_i}^{m_{f_i}} m_{g_i}^{m_{g_i}}}{\mathbb{E}[|f_i|^2]^{m_{f_i}}\mathbb{E}[|g_i|^2]^{m_{g_i}}\Gamma(m_{f_i})\Gamma(m_{g_i})} \\ &\times \int_0^\infty y^{m_{g_i}-m_{f_i}-1} e^{-\left(\frac{m_{f_i}\gamma\mathbb{E}[|g_i|^2]}{y\bar{\gamma}_i} + \frac{m_{g_i}y}{\mathbb{E}[|g_i|^2]}\right)} dy \end{aligned} \quad (3)$$

Thus, the PDF of γ_i can be found by solving the integral in (3) using [3, eqn. (3.471)], yielding

$$p_{\gamma_s}(\gamma) = \frac{2\left(\frac{\gamma m_{f_i}m_{g_i}}{\gamma_i}\right)^{m_{f_i}+m_{g_i}/2}}{\gamma\Gamma(m_{f_i})\Gamma(m_{g_i})}K_{m_{g_i}-m_{f_i}}\left(2\sqrt{\frac{m_{f_i}m_{g_i}\gamma}{\bar{\gamma}_i}}\right) \quad (4)$$

where $K_i(x)$ is the modified Bessel function of the second kind of order i. Recalling the independence of h, f_i, g_i, the MGF of γ_d, i.e. $M_d(-s) = \mathbb{E}\{e^{-s\gamma d}\}$, can be written as

$$M_d(-s) = M_h(-s)\prod_{i=1}^{R} M_i(-s) \quad (5)$$

where $M_h(-s)$ and $M_i(-s)$ are the MGF of the γ_h and γ_i, respectively. It can be easily shown that $M_h(-s) = \left(1 + \frac{\bar{\gamma}_h}{m_h}s\right)^{-m_h}$ where the m_h are the Nakagami-*m* fading parameters of h and $\bar{\gamma}_h = \mathbb{E}[|h|^2]\frac{\varepsilon_s}{N_0}$ Considering (4), and with the help of [3, eqn. (6.631)], which once used with some extra manipulations, the MGF of γ_i can be calculated as

$$M_i(-s) = \delta e^{m_{f_i}m_{g_i}/2\bar{\gamma}_i s} s^{1-m_{f_i}-m_{g_i}/2} W_{1-m_{f_i}-m_{g_i}/2,\, m_{g_i}-m_{f_i}/2}\left(\frac{m_{f_i}m_{g_i}}{2\bar{\gamma}_i s}\right) \quad (6)$$

where $W_{a,b}(x)$ is Whittaker's function of orders a and b [3, Subsection 9.224], and

$$\delta = \left(\frac{m_{f_i}m_{g_i}}{\bar{\gamma}_i}\right)^{m_{f_i}+m_{g_i}-1/2}\frac{\Gamma(m_{g_i}/2)\Gamma(m_{f_i}/2)}{\Gamma(m_{f_i})\Gamma(m_{g_i})} \quad (7)$$

Let $\rho \triangleq \sin^2(\frac{\pi}{M})$. Using $M_d(-s)$, the error rate can be evaluated for M-PSK as [4, p. 271]

$$P_e(R) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} M_d\left(\frac{-\rho}{\sin^2\phi}\right)d\phi \quad (8)$$

Simulation results: We assume $\mathbb{E}[|f_i|^2] = \mathbb{E}[|g_i|^2] = \mathbb{E}[|h|^2]$, and $m_{f_i} = m_{g_i} = m_h = m = 1$. We use the distributed space–time codes studied in [2], and the signal symbols are modulated as QPSK. Fig. 1 confirms that the analytical results attained in the previous Section for finding SER have the same performance as practical full-diversity distributed space–time codes. Assuming that Gray coding is used, BER becomes SER divided by $\log_2(M)$. Two values of $m = 0.5$, 2 are used, and curves are corresponding to different number of relays $R = 1, 2, 3$.

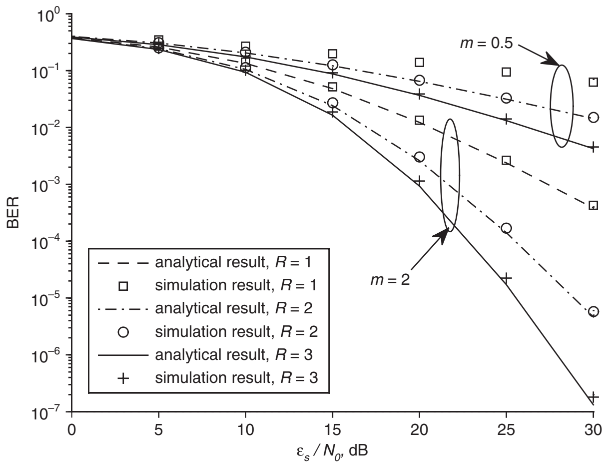

Fig. 1 *Average SER curves of relay networks employing AF DSTC and QPSK signals over Nakagami-m channels*

Conclusion: Performance analysis for AF DSTC networks over independent non-identical Nakagami-*m* fading channels have been investigated. Our numerical results show that the derived symbol error rate is in accordance with simulation results.

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