Performance Analysis of Repetition-Based Cooperative Networks with Partial Statistical CSI at Relays

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***Abstract*—This letter analyzes the performance of repetition-based cooperative diversity wireless networks using *amplify-and forward* relaying, in which each relay has only statistical knowledge of the source-relay link. The network channels are modeled as independent, non-identical, Rayleigh distributed coefficients. The exact symbol error rate is derived using the moment generating function (MGF). We derive the probability density function and MGF of the total SNR. Then, the MGF is used to determine the symbol error rate (SER). The diversity order of the amplify-and-forward cooperation with partial statistical channel state information is also found via the asymptotic behavior of the average SER, and it is shown that the cooperative network achieves full diversity. Our analytical results are confirmed by simulations.**

***Index Terms*—Wireless relay network, performance analysis, cooperative communications.**

I. Introduction

COOPERATIVE diversity networks technique is a promising solution for the high data-rate coverage required in future cellular and ad-hoc wireless communications systems. Several cooperation strategies with different relaying techniques, including amplify-and-forward (AF), decode-and-forward (DF), and selective relaying, have been studied in Laneman et al.'s seminal paper [1]. Cooperative transmissions have been categorized into space-time coded cooperation (see, e.g., [2]–[5]) and repetition-based cooperation (see, e.g., [6]–[8]).

In [7], the authors derived asymptotic average symbol error rate (SER) for AF cooperative networks, when relays have *instantaneous* channel state information (CSI). An exact average SER analysis for a AF cooperative network (with knowledge of instantaneous CSI at the relays) is presented in [6]. Using scaling factors which depend on partial statistical CSI in AF relays, i.e., each relay has only statistical knowledge of the source-relay channel, were first proposed in [9]. In [3], [10], [11], relays with partial statistical CSI are used in space-time coded cooperation.

In this letter, we apply AF relaying with partial statistical CSI to the case of repetition-based cooperation, in which R amplifying relays retransmit the source's signal in a time-division multiple-access (TDMA) manner. We derive the exact average SER of this system for M-PSK transmissions over Rayleigh-fading channels. We first find closed-form expressions for the cumulative distribution function (CDF), probability density function (PDF) and MGF of the total SNR. Then, the MGF is used to determine the average SER. We also derive the diversity order of the AF cooperation with partial statistical CSI via the asymptotic behavior of the average SER, and show that the cooperative network presented in this paper achieves full diversity order $R+1$.

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II. System Model

Consider a network consisting of a source, R relays, and one destination. We denote the source-to-destination, source-to-ith relay, and ith relay-to-destination links by f_0, f_i, and g_i, respectively. Suppose each link has Rayleigh fading, independent of the others. Therefore, f_0, f_i, and g_i are complex Gaussian random variables, which are jointly independent, with zero-mean and variances $\sigma^2_{f_0}$, $\sigma^2_{f_i}$, and $\sigma^2_{g_i}$, respectively. Similar to [1], our scheme requires two phase of transmission. During the first phase, the source node transmits a signal $s(n)$, where n is the time index, to *all* relays and the destination. The received signal at the destination and the ith relay from the source in the first phase can be written as

$$y_0(n) = \sqrt{\varepsilon_0} f_0 s(n) + w_0(n), \tag{1}$$

$$r_i(n) = \sqrt{\varepsilon_0} f_i s(n) + v_i(n), \tag{2}$$

respectively, where ε_0 is the average total transmitted symbol energy of the source, since we assume the information bearing symbols $s(n)$'s are normalized to one (M-PSK), and $w_0(n)$ and $v_i(n)$ are complex zero-mean white Gaussian noises with variances N_0 and N_i, respectively. Under repetition-based amplify-and-forward cooperation, each relay scales its received signal with the scaling factor α_i. Then, the ith relay retransmits the scaled version of the received signal towards the destination in the ith interval of the second phase, i.e.,

$$y_i(n) = g_i\, \alpha_i\, r_i(n) + w_i(n), \tag{3}$$

where $w_i(n)$ is a complex zero-mean white Gaussian noise with the variance of N_0. When there is no instantaneous CSI at the relays, but statistical CSI of the source-to-ith relay link is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is,

$$\alpha_i = \sqrt{\frac{\varepsilon_i}{\varepsilon_0 \sigma^2_{f_i} + N_i}}, \tag{4}$$

where ε_i is the average transmitted power at relay i, such that all relays transmits with the same average power.

Assuming maximum ratio combining (MRC) at the destination, the total received signal-to-noise ratio (SNR) can be written as

$$\gamma_d = \gamma_0 + \sum_{i=1}^{R} \gamma_i, \tag{5}$$

where $\gamma_0 = |f_0|^2\varepsilon_0/N_0$ is the instantaneous SNR between the source and ith relay, and γ_i can be shown to be

$$\gamma_i = \frac{|f_i|^2|g_i|^2}{A_i|g_i|^2 + B_i}, \quad (6)$$

for $i = 1, \ldots, R$, where A_i and B_i are given by

$$A_i = \frac{N_i}{\varepsilon_0},\ B_i = \frac{N_0(\sigma_{f_i}^2\varepsilon_0 + N_i)}{\varepsilon_0\varepsilon_i}. \quad (7)$$

III. Performance Analysis

A. Exact Symbol Error Probability Expression

In this subsection, we will derive the SER formula of a repetition-based relay network with noncoherent relays, using the MGF method. First, we will derive the PDF of γ_i in (6). This is needed for calculating the average SER.

Using the same procedure for calculating the CDF of product of two exponential random variable which is given in [3, Eq. (19)], and with the help of [12, Eq. (3.324)], the CDF of γ_i in (6) can be presented to be

$$\Pr\{\gamma_i < \gamma\} = 1 - 2\, e^{-\frac{A_i\gamma}{\sigma_{g_i}^2}} \sqrt{\frac{B_i\gamma}{\sigma_{f_i}^2\sigma_{g_i}^2}} K_1\left(2\sqrt{\frac{B_i\gamma}{\sigma_{f_i}^2\sigma_{g_i}^2}}\right), \quad (8)$$

where $K_j(x)$ is the modified Bessel function of the second kind of order j [13]. Thus, the PDF of γ_i can be found by taking the derivative of (8) with respect to γ, and using [14, Eq. (24.55)], yielding

$$p(\gamma_i) = \frac{2B_i}{\sigma_{f_i}^2\sigma_{g_i}^2} e^{-\frac{A_i\gamma_i}{\sigma_{g_i}^2}} K_0\left(2\sqrt{\frac{B_i\gamma_i}{\sigma_{f_i}^2\sigma_{g_i}^2}}\right) + \frac{2A_i}{\sigma_{g_i}^2}\sqrt{\frac{B_i\gamma_i}{\sigma_{f_i}^2\sigma_{g_i}^2}} e^{-\frac{A_i\gamma_i}{\sigma_{g_i}^2}} K_1\left(2\sqrt{\frac{B_i\gamma_i}{\sigma_{f_i}^2\sigma_{g_i}^2}}\right). \quad (9)$$

Recalling the independence of f_0, f_i, and g_i, the MGF of γ_d in (5), i.e., $M_d(-s) = \mathbb{E}\{e^{-s\gamma_d}\}$, can be written as

$$M_d(-s) = M_0(-s)\prod_{r=1}^{R} M_i(-s), \quad (10)$$

where $M_i(-s)$ is the MGF of γ_i in (6), while $M_0(-s) = \frac{1}{1+\sigma_{f_0}^2\varepsilon_0 s/N_0}$ is the MGF of γ_0. Considering (9), and with the help of [12, Eq. (6.643)], the MGF of γ_i can be calculated as

$$M_i(-s) = \left[\sqrt{\frac{B_i}{(s+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}} W_{-\frac{1}{2},0}\left(\frac{B_i}{(s+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}\right) + \frac{A_i\Gamma(2)}{\sigma_{g_i}^2(s+\frac{A_i}{\sigma_{g_i}^2})} W_{-1,-\frac{1}{2}}\left(\frac{B_i}{(s+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}\right)\right] e^{\frac{B_i}{2(s+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}}, \quad (11)$$

where $W_{a,b}(x)$ is Whittaker's function of orders a and b (see e.g., [13] and [12, Subsection 9.224]). Note that Whittaker's function can be presented in terms of a generalized hypergeometric function (see, e.g., [12, Eq. (9.220)]).

Using $M_d(-s)$, the the average SER for M-ary phase-shift keying (M-PSK) can be written as [15, p. 271]

$$P_e(R) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_d\left(\frac{-\rho}{\sin^2\phi}\right) d\phi, \quad (12)$$

where $\rho = \sin^2\left(\frac{\pi}{M}\right)$. The result of (12) can also be used to find average SER for nonconstant modulus transmissions like M-ary amplitude modulation (M-AM and M-QAM as indicated in [15, Eqs. (9.19) and (9.21)].

B. Diversity Analysis

In this subsection, we will study the achievable diversity gain in a AF repetition-based network containing R relays with partial statistical CSI at the relays.

A tractable definition of the diversity, or diversity gain, is $G_d = -\lim_{\mu\to\infty}\frac{\log(P_e(R))}{\log(\mu)}$, where μ is the SNR [16, Eq. (1.19)]. Furthermore, (12) can be upper-bounded by $P_e(R) \le \left(1-\frac{1}{M}\right)M_d(-\rho) < M_d(-\rho)$ [15, p. 271]. Hence, we have

$$G_d \ge -\lim_{\mu\to\infty}\frac{\log(M_d(-\rho))}{\log(\mu)}. \quad (13)$$

Therefore, using (10), we can find the diversity order of repetition-based AF system in which relays have partial statistical CSI as follows:

$$G_d \ge -\lim_{\mu\to\infty}\frac{\log(M_d(-\rho))}{\log(\mu)} = -\lim_{\mu\to\infty}\frac{\sum_{k=0}^{R}\log(M_k(-\rho))}{\log(\mu)} = -\sum_{k=0}^{R}\lim_{\mu\to\infty}\frac{\log(M_k(-\rho))}{\log(\mu)}. \quad (14)$$

For the simplicity in deriving the expressions and without lose of generality, similar to [11], we assume equal power allocation among the two transmission phases and among the relays, i.e., $\varepsilon_0 = R\,\varepsilon_i$, for $i = 1, \ldots, R$, and $N_i = N_0$. Then, by defining $\mu = \frac{N_0}{\varepsilon_0}$, we have

$$\lim_{\mu\to\infty}\frac{-\log(M_0(-\rho))}{\log(\mu)} = \lim_{\mu\to\infty}\frac{\log\left(1+\sigma_{f_0}^2\mu\rho\right)}{\log(\mu)} = 1. \quad (15)$$

Now, for calculating (14) we need to solve $\lim_{\mu\to\infty}\frac{\log(M_i(-\rho))}{\log(\mu)}$, for $i = 1, \ldots, R$.

With the help of the integral in [12, Eq. (9.22)], it can be shown that $W_{\beta,\frac{1}{2}+\beta}(x) = x^{-\beta}e^{x/2}\Gamma(2\beta+1, x)$, where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [12, Eq. (8.350)]. On the other hand, the incomplete gamma function can be represented as exponential integrals $E_n(x) = x^{n-1}\Gamma(1-n, x)$, where n is an integer [13, Eq. (5.1.45)]. Thus, we can represent $W_{-\frac{1}{2},0}(\cdot)$ and $W_{-1,-\frac{1}{2}}(\cdot)$ in terms of exponential integrals, $E_n(\cdot)$:

$$W_{-\frac{1}{2},0}(x) = x^{\frac{1}{2}}e^{\frac{x}{2}}E_1(x),\ W_{-1,-\frac{1}{2}}(x) = e^{\frac{x}{2}}E_2(-x). \quad (16)$$

Therefore, $M_i(-\rho)$ in (11) can be rewritten as

$$M_i(-\rho) = e^{\frac{B_i}{(\rho+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}}\left[\frac{B_i}{(\rho+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}E_1\left(\frac{B_i}{(\rho+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}\right) + \frac{A_i\Gamma(2)}{\sigma_{g_i}^2(\rho+\frac{A_i}{\sigma_{g_i}^2})}E_2\left(\frac{B_i}{(\rho+\frac{A_i}{\sigma_{g_i}^2})\sigma_{f_i}^2\sigma_{g_i}^2}\right)\right]. \quad (17)$$

Furthermore, using [13, Eq. (5.1.11)], the series representation of $E_i(x)$, for $x > 0$, can be expressed as

$$E_1(x) = -\kappa - \log(x) - \sum_{k=1}^{\infty}\frac{-x^k}{k\,k!}, \quad (18)$$

$$E_2(x) = e^{-x} + \kappa\,x + x\log(x) + \sum_{k=1}^{\infty}\frac{(-x)^{k+1}}{k\,k!}. \quad (19)$$

where κ is Euler's constant, i.e., $\kappa \approx 0.5772156$ [12]. Then, using (18) and (19), and by defining $C_i = \frac{R}{\mu\rho\,\sigma_{g_i}^2}$, we have

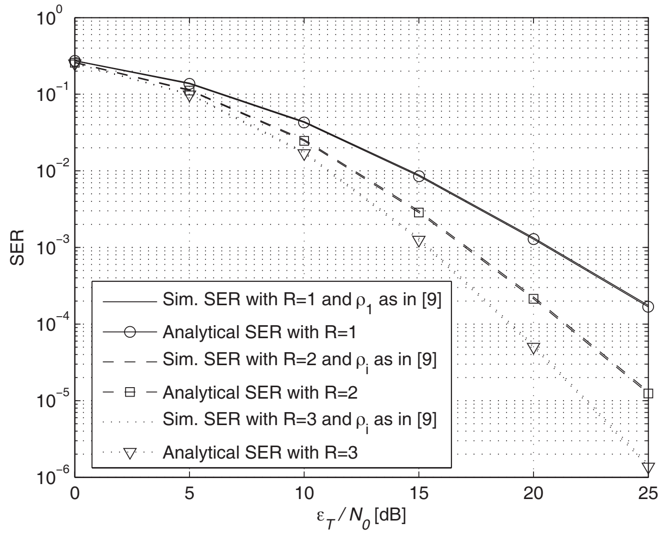

Fig. 1. The average SER curves of relay networks employing repetition-based transmission with scaling factor in (4) and BPSK signals.

$$\begin{aligned}
&\lim_{\mu\to\infty} \frac{-\log\left(M_i\left(-\rho\right)\right)}{\log\left(\mu\right)} \\
&= \lim_{\mu\to\infty} \frac{-\log\left(e^{C_i}\left[C_i E_1(C_i) + \frac{2A_i\Gamma(2)}{\sigma_{g_i}^2 g} E_2(C_i)\right]\right)}{\log\left(\mu\right)} \\
&= \lim_{\mu\to\infty} \frac{-\log\left(e^{C_i} C_i E_1(C_i)\right)}{\log\left(\mu\right)} \\
&= \lim_{\mu\to\infty} -\frac{C_i + \log\left(C_i\right) + \log\left(-\log\left(C_i\right)\right)}{\log\left(\mu\right)} = 1. \qquad (20)
\end{aligned}$$

In the last equation, we have used the l'Hôpital's rule. As a result, by substituting (15) and (20) in (14), we find that the system provides full diversity, i.e., $G_d = R + 1$.

IV. Simulation Results

In this section, we show numerical results of the analytical SER for binary phase shift keying (BPSK) modulation. We plot the performance curves in terms of average SER versus SNR of the transmitted signal (ε_T/N_0), where ε_T is the total transmitted power during two phases, i.e., $\varepsilon_T = \varepsilon_0 + \sum_{i=1}^{R} \varepsilon_i$. We use the block fading model and it is assumed that the relays and the destination have the same value of noise power ($N_i = N_0$). We assume all the source-relays and relays-destination links have unit-variance Rayleigh flat fading, i.e., $\sigma_{f_i}^2 = \sigma_{g_i}^2 = 1$, and the direct source-destination has doubled distance as source-relays link, which by assuming path loth exponent 2, $\sigma_{f_0}^2 = 1/4$. Furthermore, we assume equal power allocation scheme, i.e., $\varepsilon_0 = R\,\varepsilon_i$, for $i = 1, \ldots, R$, which is a reasonable choice [7], [10].

Fig. 1 confirms that the analytical SER expressions in Subsection III-A for finding the average SER have similar performance as simulation result. We consider a network with $R = 1, 2, 3$ and we have averaged the error rate over 200 000 fading realization. The analytical results are based on (12).

V. Conclusion

Performance analyzes for AF cooperative networks with noncoherent relays over independent, non-identical, Rayleigh fading channels has been investigated. The closed-form expressions for the CDF, pdf, and MGF of the total received SNR at the destination have been derived. Then, we have computed the exact SER of a repetition-based cooperative network with R parallel relays and M-PSK signaling using the gain in (4). Using the asymptotic analysis of the SER expression, we have shown that this cooperative network achieves full diversity order $R + 1$. Simulations are in accordance with analytic results.

References

[1] J. N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[2] R. U. Nabar, H. Bölcskei, and F. W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[3] B. Maham and A. Hjørungnes, "Distributed GABBA space-time codes in amplify-and-forward cooperation," in *Proc. IEEE Information Theory Workshop (ITW'07)*, Bergen, Norway, July 2007, pp. 189–193.

[4] K. Seddik, A. Sadek, A. Ibrahim, and K. Liu, "Design criteria and performance analysis for distributed space-time coding," *IEEE Trans. Veh. Technol.*, to appear, 2008.

[5] T. Wang, Y. Yao, and G. B. Giannakis, "Non-coherent distributed space-time processing for multiuser cooperative transmissions," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3339–3343, Dec. 2006.

[6] P. A. Anghel and M. Kaveh, "Exact symbol error rate probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416–1421, Sept. 2004.

[7] A. Ribeiro, A. Cai, and G. B. Giannakis, "Symbol error probablity for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1264–1273, May 2005.

[8] T. Wang, A. Cano, G. B. Giannakis, and J. N. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Trans. Commun.*, vol. 55, no. 6, pp. 830–841, Apr. 2007.

[9] D. Chen and J. N. Laneman, "Cooperative diversity for wireless fading channels without channel state information," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, Monterey, CA, Nov. 2004.

[10] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

[11] Y. Jing and H. Jafarkhani, "Using orthogonal and quasi-orthogonal designs in wireless relay networks," *IEEE Trans. Inform. Theory*, vol. 53, no. 11, pp. 4106–4118, Nov. 2007.

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, USA: Academic, 1996.

[13] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover Publications, 1972.

[14] M. R. Spiegel and J. Liu, *Schaum's Mathematical Handbook of Formulas and Tables*. McGraw-Hill, 1998.

[15] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York: Wiley, 2000.

[16] H. Jafarkhani, *Space-Time Coding Theory and Practice*. Cambridge, UK: Cambridge Academic Press, 2005.