Interference Analysis for Spatial Reused Cooperative Multihop Wireless Networks

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Abstract—We consider a decode-and-forward based wireless multihop network with a single source node, a single destination node, and \(N\) intermediate nodes. To increase the spectral efficiency and energy efficiency of the system, we propose a cooperative multihop communication with spatial reuse, in which interference is treated as noise. The performance of spatial-reused space-time coded cooperative multihop network is analyzed over Rayleigh fading channels. More specifically, the exact closed-form expression for the outage probability at the \(n\)th receiving node is derived when there are multiple interferences over non-i.i.d. Rayleigh fading channels. Moreover, in high SNR scenario, closed-form asymptotic formulas for the outage probability are derived, from which, we show that the full-spatial diversity is still achievable given interferences from the transmission of concurrent packets. In addition, we propose a simple power control scheme which is only dependent on the statistical knowledge of channels. Finally, the analytic results were confirmed by simulations. It is shown by simulations that the spatial-reused multihop transmission outperforms the interference-free multihop transmission in terms of energy efficiency in low and medium SNR scenarios.

I. INTRODUCTION

Cooperative multihop wireless systems have been considered as the promising technique to extend coverage area and reduce power consumption [1], [2]. In [2], three cooperative multihop transmission protocols were proposed that compromise between spectral and energy efficiencies. To further increase of multiplexing gain and energy efficiency, in this paper, we consider a cooperative multihop transmission with interference due to the simultaneous transmission of multiple packets. The idea of multihop transmission with spatial reuse is proposed in [3]. To facilitate concurrent transmission of several packets in the network, the available bandwidth is reused among transmitters, with a minimum division of \(K\) nodes between simultaneously transmitting nodes. Therefore, we have to deal with a type of co-channel interference (CCI).

The performance analysis of multihop transmission in Rayleigh fading channels under CCI were recently studied in the number of literature such as [4]–[7]. However, to the best of our knowledge, this is the first work that investigate the performance analysis of multihop networks with multiple interferences over non-i.i.d. Rayleigh fading. This is of primarily importance for the study of interference due to spatial reused cooperative multihop transmission.

In this paper, we study the performance analysis of the decode-and-forward based cooperative multihop transmission with interference due to the concurrent transmission of multiple data. We show that the capacity of the cooperative multihop transmission can be improved by using spatial reuse scheme. The achievable rate of the multihop transmission can be increased up to \(\left\lfloor \frac{N+1}{K} \right\rfloor\) times, where \(K\) is the minimum separation of concurrently transmitting nodes in a network with \(N\) relays, in expense of performance degradation. Moreover, we derive a closed-form expression for the outage probability of the cooperative multihop system in presence of interferences due to the spatial reuse over Rayleigh fading channels. The simplicity of the calculated expression can give insights on performance of the system and ways to optimize the system. In addition, the asymptotic formulas for different signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) conditions are derived. Furthermore, we formulate the problem of minimizing the transmit power for an outage-restricted equal power multihop network under the assumption of no instantaneous CSI knowledge at the transmitters.

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II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

Consider a wireless communication network in which the source \( s \) intend to transmit its data to the destination \( d \) with the help of \( N \) cascaded intermediate nodes. Due to the broadcast nature of the wireless channel, some intermediate relays can overhear and retransmit the received packets. The channel between any two nodes in the network is assumed to be a Rayleigh fading. Similar to [1], each transmission could be either a broadcast transmission where one node transmits the signal that is heard by multiple receivers, or a cooperative transmission where multiple nodes concurrently transmit the signal to a single receiving node. Here, we employ the cooperation protocol proposed in [8] where consists of \( N + 1 \) transmission phases. We assume there is no CSI knowledge at transmitters and only statistical CSI is available at the transmitters. Thus, distributed space-time coded transmissions like codes proposed in [9] is a feasible method to be employed for the cooperative transmission.

The transmission protocols have two major phases of non-cooperative and cooperative stages. Depending on the requirements, non-cooperative phase may contain one or multiple steps. The next phases employ space-time cooperated transmission. As an example, Fig. 1 depicts a protocol employing distributed quasi-orthogonal space time code (D-QOST) with \( M = 4 \), where \( M \) is the number of cooperating nodes. The detailed description of cooperative multihop protocols, i.e. Broadcast-then-Coopeate, Multi-hop-then-Cooperate, and Full-Cooperation, is studied in [10]. To be consistent through the paper, from now on, we consider the Multi-hop-then-Cooperate protocol illustrated in Fig. 1. However, the proposed procedure can be easily modified using two other protocols. Assuming the usage of full-rate distributed space-time codes, the number of cooperating nodes is equal to the transmitting packets. Hence, the source node intends to transmit \( M \) packets to the destination. The signals transmitted by the source terminal during the \( n \)-th time slot of Phase 1 is denoted as \( s_m(t) \), \( m = 1, \ldots, M \) where \( t \) is the time index and is indicated as a group of \( M \) packets transmitted at a given time, and \( E[s_m(t)] = 0 \) and \( E[s_m(t)^2] \) is the average transmit power of node \( n \). In Phase 1, the source transmits the information, and the signal received at the \( n \)-th node in the first \( M \) time slots is given by

\[
y_{n,m}(t) = \sqrt{P_{0,1}} h_{0,i} s_m(t) + v_{i,m}(t), \quad \text{for } m = 1, \ldots, M,
\]

where \( P_{0,1} \) is the average transmit power of the source symbol in the first phase, and \( v_{i,m} \) denotes complex zero-mean white Gaussian noise with variance \( \mathcal{N}_0 \). The link coefficients from the \( j \)-th node to the \( n \)-th node \( h_{j,n} \), \( j = 0, 1, \ldots, N \), \( i = 1, 2, \ldots, N + 1 \) are complex Gaussian random variables with zero-mean and variances \( \sigma^2_{j,i} \), where the \( N + 1 \)-th node is the destination \( d \). We assume coherence times of the channels are such that channel coefficients \( h_{j,i} \) are not varying during \( M \) consecutive time slots. Note that \( \sigma^2_{j,i} = (d_0/d_{j,i})^\delta \) is the path-loss coefficient, where \( d_{j,i} \) is the distance between nodes \( j \) and \( i \), \( \delta \) is the path-loss exponent, which typically lies in the range of 2 \( \leq \delta \leq 6 \), and \( d_0 \) depends on the operating frequency.

For the first \( M - 1 \) phases, we repeat the non-cooperative transmission described above for the \( n \)-th relay, \( 1 \leq n < M - 1 \) to retransmit the source data. After initial stages, the cooperative transmission is utilized for routing the source packets to the destination. In contrast to [1] where transmitters are able to modify their phases, here, the instantaneous CSI is not known at the transmitter nodes. This assumption is realistic for most wireless systems. Hence, space-time coded cooperation is the appropriate choice to achieve the spatial diversity gain. In Phase \( n \), \( M \leq n \), the previous \( M \) nodes transmit their signals concurrently toward the next node using an appropriate distributed space-time code.

To facilitate simultaneous transmission of several packets in the network, the available bandwidth is reused among transmitters, with a minimum separation of \( K \) nodes between concurrently transmitting nodes. Fig. 2 shows a spatial-reused cooperative multihop network with \( M = 2 \). Since in Phase \( n \geq M \), \( M \) nodes transmit the intended data to the \( n \)-th node, and by assuming half-duplex transmission, the minimum value of the spatially-reused factor \( K \) is \( M + 1 \), and thus, \( M + 1 \leq K \leq N + 1 \). For the message detection, the \( n \)-th node intend to receive signals not coming from the \( M \) previous nodes as Gaussian interference. In this work, we treat the interference as additive Gaussian noise. In the presence of inter-network interference from the spatial-reused nodes, for \( n \geq M \) the received signal at the \( n \)-th receiving node can be represented as

\[
y_{n,m}(t) = \sum_{m=1}^{M} \sqrt{P_{n-m,m}} h_{n,m} x_{n,m}(t)
\]

where \( x_{n,m}(t) \) is the zero-mean space-time coded signal, normalized as \( E[x_{n,m}(t)^2] = 1 \) during the whole packet transmission, and \( P_{n-m,m}, \ m = 1, \ldots, M, \) is the average transmit power of node \( n - m \) during the \( n \)-th time slot of Phase \( n \). In (2), \( U_n \) denotes the set of nodes transmitting simultaneously with Nodes \( n - m \), \( m = 1, \ldots, M \), due to spatial reuse, i.e.,

\[
U_n = \{ u \in \{1, 2, \ldots, N + 1 \} \mid u \neq n \text{ and } K \text{ divides } n - u \},
\]

and \( \tau = \frac{n - u}{K} \). It is assumed that all hops use the total bandwidth \( W \), and we are interested in the reliable delivery of
messages at a rate of $R$ bits/second/Hertz by consuming the minimum total transmit power.

III. PERFORMANCE ANALYSIS OF SPATIAL REUSED COOPERATIVE MULTIHOP TRANSMISSION

In the following, the outage probability $\rho_{\text{out}} \triangleq \Pr\{r_n < R\}$ of the $n$th receiving node at the $n$th hop in spatial-reused system is derived, which describes the probability that the transmit rate $R$ is larger than the supported rate $r_n$. This probability depends on the fixed transmission parameters and the channel condition within the hops.

In the cooperative multihop transmission with spatial reuse factor of $K$, from (1) and (2), the instantaneous achieved rate at the $n$th hop becomes

$$r_n = \frac{1}{K} \log \left( 1 + \frac{\sum_{m=1}^{M} P_{n-m,n} |h_{n-m,n}|^2}{N_0 W + \sum_{u \in U_n} \sum_{m=1}^{M} P_{u-m,u} |h_{u-m,u}|^2} \right),$$

where $P_{n-i,n} = 0$ for $n = 1, \ldots, M - 1, i = 2, \ldots, M$.

A. Outage Probability

Now, we calculate an exact closed-form expression for the outage probability at the $n$th receiving node in presence of interference from the multiple-antenna secondary BS. By defining $\gamma_{\text{th}} \triangleq 2 R K - 1$, the outage probability of non-cooperative transmission can be represented as

$$\rho_{\text{out}} = \Pr \left\{ \sum_{m=1}^{M} P_{n-m,n} |h_{n-m,n}|^2 < \gamma_{\text{th}} \right\}.$$

Thus, the receiver can reliably decode the source data whenever $r_n \geq R$. For decoding the message correctly, the outage probability must be less than a desired end-to-end outage probability $\rho_{\text{max}}$.

Lemma 1: Considering a set of independent exponential random variables $X = \{X_1, \ldots, X_M\}$ with mean of $\sigma^2 x_m$, $m = 1, \ldots, M$, the cumulative distribution function (CDF) of the summation of independent-not-identical exponentially distributed random variables, i.e., $X = \sum_{m=1}^{M} X_m$ is given by

$$\Pr\{X < x\} = \sum_{m=1}^{M} \alpha_m \left(1 - e^{-x \sigma^2 x_m}\right).$$

where

$$\alpha_m = \prod_{j=1}^{M} \frac{\sigma^2 x_m}{\sigma^2 x_j}.$$

Proof: The proof is given in [11].

Using the inductive reasoning, the following lemma can be obtained:

Lemma 2: For $\alpha_m$ defined in (7), the following properties hold:

$$\sum_{m=1}^{M} \alpha_m = 1,$$

where $\Omega_M(\nu) = \sum_{i=1}^{M} \Omega_M(\nu - i)$. Note that for the case of $M = 2$, $\{\Omega_2(\nu)\}$ is a Fibonacci sequence, i.e., $\Omega_2(\nu) = \Omega_2(\nu + 1) + \Omega_2(\nu - 2)$.

Proposition 1: Given finite sets of independent random variables $X' = \{X_1, \ldots, X_M\}$ and $Y' = \{Y_1, \ldots, Y_Q\}$ with non-identical exponential distribution and mean of $\sigma^2 y_q$, $q = 1, \ldots, Q$, respectively, the CDF of

$$\text{CDF} = \frac{\sum_{m=1}^{M} X_m}{1 + \sum_{q=1}^{Q} Y_q}$$

can be calculated as

$$\Pr\{\text{CDF} < \gamma\} = 1 - \sum_{m=1}^{M} \alpha_m e^{-\sigma^2 y_q \gamma\gamma + 1} \prod_{j=1}^{M} \left(\frac{\sigma^2 y_q}{\sigma^2 y_j}\right)^{-1}.$$

Proof: The proof is given in Appendix I.
the relationship between the end-to-end outage probability of $P_{\text{des}} = p_{N+1}$ and $p_0$, we have

$$P_{\text{des}} = 1 - \sum_{v=0}^{N-1} (1 - p_0)^{\Omega_m(v)} = 1 - (1 - p_0)^{\sum_{v=0}^{N-1} \Omega_m(v)}. \quad (15)$$

Thus, the target outage probability at each hop $\rho_0$ can be represented in terms of the desired probability of error at the destination $\rho_{\text{des}}$.

Furthermore, assuming $\rho_{\text{out}} \ll 1$, the outage probability at the destination in (14) can be approximated as follows:

$$\rho_{\text{out}} \approx \sum_{\nu=0}^{N} \Omega(\nu) \rho_{\text{out}}^{N-\nu+1}. \quad (16)$$

IV. POWER ALLOCATION FOR MULTIHOP TRANSMISSION WITH SPATIAL-REUSE INTERFERENCE

In the previous section, the impact of spatial-reuse interference on the performance of cooperative multihop system is studied. Here, we introduce a power control strategy to improve the system performance. In this section, we derive the required power for the multihop transmission scheme discussed in Section II in order to achieve a certain rate $R$ with a given outage probability QoS. Finding the optimum value of the transmit powers is complicated due to the complexity of outage probability $\Pr(\{r_n < R\})$ derived in (12). However, by assuming an equal power in every nodes, in the following, a suboptimal power allocation strategy is proposed.

In the case of interference-free transmission, as stated in [10, Theorem 1], the cooperative transmit power coefficients should be equal in each transmission phase. To get a more accurate result, we further assume equal transmission power in all phases to achieve a target outage probability QoS. Thus, assuming the equal transmit power, i.e., $P_{n-m,n} = P_{u-m,u}$ for $m = 1, \ldots, M$, $n = 1, \ldots, N$, and $u \in U_n$, we have

$$P_{\text{out}}^{\text{n}} = 1 - e^{-\frac{n N \Omega u}{\sigma_{n-m,n}^2}} \prod_{u \in U_n} \prod_{i \in M_u} \left( \frac{\sigma_{n-m,n}^2}{\sigma_{n-i,n}^2} \right) \rho_{\text{th}}^{-1}, \quad (17)$$

for $n = 1, \ldots, M - 1$, where $M_u = \{1\}$ if $u < M$, and $M_u = \{2, \ldots, M\}$, if $u \geq M$. For $n = M, \ldots, N + 1$, the outage probability can be rewritten as

$$P_{\text{out}}^{\text{n}} = 1 - \sum_{m=1}^{M} A_{m,n} e^{-\frac{n N \Omega u}{\sigma_{n-m,n}^2}}, \quad (18)$$

where

$$A_{m,n} = \frac{M}{\prod_{j \neq m} \sigma_{n-m,n}^2 - \sigma_{n-j,n}^2} \prod_{u \in U_n} \prod_{i \in M_u} \left( \frac{\sigma_{n-m,n}^2}{\sigma_{n-i,n}^2} \rho_{\text{th}}^{-1}. \right) \quad (19)$$

Since $P_{\text{out}}^{\text{n}}$ is an decreasing function of the power coefficient $P_n$ for $P_n \geq 0$, to find the minimum value of the problem in $P_n$, the constraint $P_{\text{out}}^{\text{n}} \leq \rho_n$ is turned into the equality. Thus, the positive root of $P_{\text{out}}^{\text{n}} - \rho_n = 0$ should be calculated. Hence, from (17), for $n = 1, \ldots, M - 1$, we have

$$P_n = \frac{-\gamma_n N \Omega u}{\sigma_{n-m,n}^2} e^{-\frac{\gamma_n N \Omega u}{\sigma_{n-m,n}^2}} \frac{(\rho_n - 1)}{\ln(1 - \rho_n) + \sum_{u \in U_n} \sum_{i \in M_u} \ln \left( \frac{\rho_n}{\sigma_{n-i,n}^2} \right) \rho_{\text{th}} + 1}. \quad (20)$$

For $n = M, \ldots, N+1$, and for a given initial value, $P_n$ can be calculated from (18) using the following recursive equation:

$$P_{n+1}^{(t+1)} = \frac{-\gamma_n N \Omega u}{\sigma_{n-m,n}^2} e^{-\frac{\gamma_n N \Omega u}{\sigma_{n-m,n}^2}} \left[ \frac{\rho_n^{(t)}}{\rho_n^{(t)}} \right] \Omega_{\text{u}} \left( N - n - 1 \right) \Omega_{\text{u}} \left( N - n + 1 \right) \prod_{n=1}^{M} A_{m,n} \prod_{n=M}^{N} - \sum_{u \in U_n} \sum_{i \in M_u} \ln \left( \frac{\rho_n^{(t)}}{\sigma_{n-i,n}^2} \right) \rho_{\text{th}} + 1 \right]. \quad (21)$$

where $P_n^{(t)}$ is the updated version of the power coefficient in the $t$-th iteration. Since $P_n^{(t)}$ is a decreasing function of $P_n$, to guarantee that $P_n^{(t)} \leq \rho_n$ where $\rho_n$ is a target outage probability per hop, we have

$$P_0 = \max \{ P_n \}. \quad (22)$$

where $P_n$ is the solution of recursive equation in (21). Assuming a fixed per-hop outage target of $\rho_n = \rho_0$ and using (15), we can represent $P_0$ in terms of ene-to-end outage probability of $\rho_{\text{des}}$ by replacing $\rho_n$ with $\rho_0 = 1 - (1 - \rho_{\text{des}})^{\sum_{\nu=0}^{N-1} \Omega_m(\nu)}$ in (21).

**Proposition 2:** In the spatial-reused multihop transmission, the minimum allowed target outage requirement at the destination is given by

$$\rho_{\text{des}} \geq 1 - \prod_{n=1}^{M} \prod_{u \in U_n} \prod_{i \in M_u} \left( \frac{\sigma_{n-m,n}^2}{\sigma_{n-i,n}^2} \rho_{\text{th}}^{-1} \right) \Omega_{\text{u}} \left( N - n + 1 \right) \prod_{n=M}^{N+1} \left( \sum_{m=1}^{M} A_{m,n} \right) \Omega_{\text{u}} \left( N - m + 1 \right) \quad (23)$$

**Proof:** The minimum amount of permissible target outage per hop can be obtained by putting $P_n \to \infty$ in (17) and (18) to get

$$\rho_n \geq 1 - \prod_{u \in U_n} \prod_{i \in M_u} \left( \frac{\sigma_{n-m,n}^2}{\sigma_{n-i,n}^2} \rho_{\text{th}}^{-1} \right) \Omega_{\text{u}} \left( N - n + 1 \right) \prod_{n=M}^{N+1} \left( \sum_{m=1}^{M} A_{m,n} \right) \Omega_{\text{u}} \left( N - m + 1 \right) \quad (24)$$

for $n = M, \ldots, N + 1$, and

$$\rho_n \geq 1 - \sum_{m=1}^{M} A_{m,n}, \quad \text{for } n = M, \ldots, N + 1. \quad (25)$$

Combining (14), (24), and (25), the minimum feasible outage probability QoS at the destination is obtained as (23).

In addition, for a given desired outage probability $\rho_{\text{des}}$ at the destination, one can find the minimum spatial-reused factor, i.e., nodes distance $K$, using Proposition 3. Moreover, it can be observed from (8), (23), and (25) that when there is no interference, we have $\rho_{\text{des}} \geq 0$, and thus, there is no limitation in choosing $\rho_{\text{des}}$.

For the case of non-cooperative multihop transmission, i.e., when $M = 1$, the closed-form solution for $P_0$ is obtained in the following proposition:

**Proposition 3:** Assuming the equal power transmission from all nodes, the minimum transmit power $P_n^\ast$ per node to achieve a per-hop outage probability of $\rho_n$ in a non-cooperative spatial-reused multihop system over Rayleigh fading channels can be expressed as
Since higher $K$ means lower concurrent transmissions, and thus, lower interferences, as $K$ increases, curves get closer to the interference-free case. Furthermore, Fig. 3 confirms the correctness of our analytical results derived in (12), since the curves are exactly match the simulations results.

**APPENDIX I**

**PROOF OF PROPOSITION 1**

The PDF of $Y_q$ is given as $p_q(y_q) = \frac{1}{\sigma_y^2} e^{-\frac{y_q}{\sigma_y^2}}$. Moreover, the CDF of $X = \sum_{m=1}^{M} X_m$ is calculated in Lemma 1. By marginalizing over the independent random variables $Y_q$, the CDF of SINR can be calculated as

$$
Pr\{\text{SINR} < \gamma\} = \int_{0}^{\gamma} \text{Pr}\{X < \gamma + \sum_{q=1}^{Q} y_q\} \prod_{q=1}^{Q} p_q(y_q) dy_q
$$

$$
= \sum_{m=1}^{M} \alpha_m \int_{0}^{\gamma} \left(1 - \frac{e^{-\frac{\gamma + \sum_{q=1}^{Q} y_q}{\sigma_m}}}{\sigma_m} \right) \prod_{q=1}^{Q} p_q(y_q) dy_q
$$

$$
= 1 - \sum_{m=1}^{M} \alpha_m e^{-\frac{\gamma}{\sigma_m}} \prod_{q=1}^{Q} \int_{0}^{\gamma} e^{-\frac{y_q}{\sigma_q}} \frac{dy_q}{\sigma_y^2},
$$

where in the third equality, we used the first property of Lemma 2 in (8). Thus, the closed-form solution for integral in (28) is obtained as (11).

**REFERENCES**


