Transmit Antenna Selection OFDM Systems with Transceiver I/Q Imbalance

Behrouz Maham, *Member, IEEE* and Olav Tirkkonen, *Senior Member, IEEE*

***Abstract*— One of the serious imperfections affecting orthogonal frequency-division multiplexing (OFDM) systems is transceiver I/Q imbalance. In this paper, the effect of I/Q imbalances on the transmit antenna selection (TAS) OFDM system is studied. The optimum antenna selection in presence of transmit and receive I/Q imbalances are proposed. We derive closed-form expressions for ergodic capacity of the system with transmit and receive I/Q imbalances. Moreover, some tight and simple lower and upper bounds for the ergodic capacity of TAS OFDM system are derived. The simplicity of the proposed formula gives insight on the performance and optimization of the system. Finally, the analytical results are confirmed by simulations.**

***Index Terms*— OFDM, multiple antenna processing, I/Q imbalance, ergodic capacity**

I. Introduction

Multiple-antenna and multiple-output (MIMO) OFDM systems, involving clever processing in both the spatial and frequency domains, have been proposed for WiFi, WiMax and fourth generation cellular systems as well as the IEEE 802.16 standard for wireless Internet access [1]. In particular, transmit antenna selection (TAS) is a low-complexity MIMO approach which selects a single transmit antenna to maximize the signal-to-noise ratio (SNR) at the receiver [2], [3]. It was shown in [4], [5] that TAS preserves the same diversity order as conventional MIMO which utilizes all the transmit antennas. This diversity order was found to be a product of the number of antennas at the source and at the destination. A low-cost implementation of such physical layers is desirable in view of mass deployment, but challenging due to impairments associated with the analog components. A major source of analog impairments in high-speed wireless communications systems is the in-phase/quadrature-phase (I/Q) imbalance [6], [7]. The I/Q imbalance is the mismatch between I and Q balances due to the analog imperfection and introduced both in the up and down conversion at the transceivers. In general, it is difficult to efficiently and entirely eliminate such imbalances in the analog domain due to power consumption, size and cost of the devices. Therefore, efficient compensation techniques in the digital baseband domain are needed for the transceivers [8], [9]. The analysis of capacity of single-antenna OFDM systems with transceiver I/Q imbalance is studied in [10]. In this paper, we find close-form solution of TAS OFDM system, which is also valid for the case of single antenna OFDM with independent channels over subcarriers.

In this paper, optimum antenna selections for multiple-input and single-output (MISO) OFDM systems with transmit or receive I/Q imbalances are found. This paper provides a comprehensive analysis of the ergodic capacity of the TAS based OFDM system with transceiver I/Q imbalance of low cost terminals. The closed-form expressions for the average capacity of the TAS OFDM system with transmit or receive I/Q imbalances are derived. Furthermore, the simple close-form formulas are calculated for different I/Q imbalance scenarios. The derived expressions give insight on performance of the system and possible ways to optimize it. The numerical results verify the tightness of the bounds.

This paper is organized as follows: Section II briefly reviews the model of a TAS OFDM system with transceiver I/Q imbalance and the corresponding antenna selection strategies. This model is considered from a statistical point of view which builds the basis for the analysis of the system average capacity in Section III. In Section IV, the overall system performance is presented via simulations and the correctness of the analytical formulas is confirmed by simulation results. Conclusions are presented in Section V.

II. Transmit Antenna Selection with Transceiver I/Q Imbalance

A. System Model

We consider an TAS OFDM system transmitting over K subcarriers with M antennas at the transmitter and a single antenna at the receiver. The system wants to transmit data symbol s_k on the k-th subcarrier for $k \in \{-K/2, -K/2+1, \dots, K/2\}$, where s_k is taken from some two-dimensional symbol constellation we assume that no data is transmitted on the direct current (DC) subcarrier and K is an even number. It is assumed that each subcarrier symbol is transmitted with equal energy E_s. Thus, the selected antenna in subcarrier k transmits with the fixed energy E_s. The impulse response of the channel is assumed shorter than the cyclic prefix. After removing the cyclic prefix, the channel for the k-th subcarrier after the Discrete Fourier Transform (DFT) can be described as a complex channel $h_{k,m}$, where m is the index of the

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Behrouz Maham is with School of Electrical and Computer Engineering, College of Engineering, University of Tehran, North Karegar, Tehran 14395-515, Iran. Olav Tirkkonen is with Department of Communications and Networking, Aalto University, and Nokia Research Center, Helsinki, Finland. Emails: `b.maham@ece.ut.ac.ir`, `olav.tirkkonen@tkk.fi`.

selected antenna. The channels $h_{k,m}$ are assumed independent Gaussian distributed random variables with variance $\sigma^2_{h_k}$, for $m = 1, \ldots, M$, due to Rayleigh fading. When analyzing Rx IQ imbalance, we assume that the length of the impulse response of the channel is of the order of the cyclic prefix (CP). Then, for $k >$ CP, $h_{k,m}$ and $\boldsymbol{h}_{-k,m}$ can be considered independent. Here, we shall assume independence, implying the tacit assumption that the channel is frequency selective, and the analysis is not valid for the subcarriers closest to DC. In [11], [12], it has been shown that I/Q imbalance of a direct-conversion transceiver in OFDM systems induces a mutual interference between subcarriers that are located symmetrically to the DC carrier. Before transmission, the baseband signal is up-converted to the radio frequency signal by using a local oscillator (LO) of the carrier frequency. Ideally, the LO outputs for the I and Q branches (representing the real part and imaginary parts) should have equal amplitudes and phase difference of $\pi/2$. However, in practice, the matching of I and Q signals is imperfect and this leads to the amplitude and phase imbalance between the I and Q signals. Such impairments are known as Tx IQ imbalance or mismatch and this severely limits the performance of the receiver, especially if cheap components or architectures, e.g., direct conversion architecture [13], are employed.

Let x_k be a complex value that is transmitted at subcarrier k. Then, the received signal on the k-th subcarrier can be written as

$$y_k = h_{k,m} x_k + w_k, \tag{1}$$

where w_k is the noise with independent and identically distributed (i.i.d.) Gaussian distribution according to $\mathcal{CN}(0, \mathcal{N}_0)$. Note that each x_k may differ from the original signal s_k at subcarrier k. Only in case of an ideal transmitter $x_k = s_k$. Likewise, each receive signal y_k may in general be different from the actually obtained signal r_k, where $r_k = y_k$ only holds for an ideal receiver. The IQ distorted signal of the kth subcarrier can be modeled using the original signal and its conjugate as

$$x_k = \alpha_{\mathrm{T}} s_k + \beta_{\mathrm{T}} s^{\dagger}_{-k}, \tag{2}$$

where $(\cdot)^{\dagger}$ represents the complex conjugate, and two complex scalers α_{T} and β_{T} are given by [6], [7]

$$\begin{aligned} \alpha_{\mathrm{T}} &= \cos\theta_{\mathrm{T}} + j\,\epsilon_{\mathrm{T}} \sin\theta_{\mathrm{T}}, \\ \beta_{\mathrm{T}} &= \epsilon_{\mathrm{T}} \cos\theta_{\mathrm{T}} - j\,\sin\theta_{\mathrm{T}}, \end{aligned} \tag{3}$$

where ϵ_{T} and θ_{T} denote the amplitude and phase imbalances between the I and Q branches of the transmitted signal, respectively. Note that since all the transmit antennas are co-located, it is a reasonable assumption that in a low cost system all transmit antennas are supported by a single local oscillator (LO), and thus, α_T and β_T do not depend on the selection of antenna m. When the matching of I and Q balances is ideal, i.e., $\epsilon_{\mathrm{T}} = 0$ and $\theta_{\mathrm{T}} = 0$, we have $\alpha_{\mathrm{T}} = 1$ and $\beta_{\mathrm{T}} = 0$. The degree of the I/Q imbalance can be evaluated in terms of a image rejection ratio (IRR), which is defined by $A_{\mathrm{T}} = \frac{|\beta_{\mathrm{T}}|^2}{|\alpha_{\mathrm{T}}|^2}$. From (1)-(2), we have

$$y_k = \alpha_{\mathrm{T}} h_{k,m} s_k + \beta_{\mathrm{T}} h_{k,m} s^{\dagger}_{-k} + w_k. \tag{4}$$

Now, consider the multiple-transmit-antenna OFDM system with I/Q imbalance at both the transmitter and receiver. Because of I/Q imbalance at the receiver, each receive symbol y_k at subcarrier k is interfered by the complex conjugated symbol $y^{\dagger}_{-k}$ received at subcarrier $-k$ and vice versa. Thus, the received signal can thus be modeled by

$$r_k = \alpha_{\mathrm{R}} y_k + \beta_{\mathrm{R}} y^{\dagger}_{-k}, \tag{5}$$

where r_k denotes the actually received symbol at subcarrier k. The complex valued weighting factors α_{R} and β_{R} of the receiver front-end are defined as

$$\begin{aligned} \alpha_{\mathrm{R}} &= \cos\theta_{\mathrm{R}} + j\,\epsilon_{\mathrm{R}} \sin\theta_{\mathrm{R}}, \\ \beta_{\mathrm{R}} &= \epsilon_{\mathrm{R}} \cos\theta_{\mathrm{R}} - j\,\sin\theta_{\mathrm{R}}, \end{aligned} \tag{6}$$

where ϵ_{R} and θ_{R} denote the amplitude and phase imbalances between the I and Q branches of the received signal, respectively. The degree of the receive I/Q imbalance can be evaluated in terms of a IRR, which is defined by $A_{\mathrm{R}} = \frac{|\beta_{\mathrm{R}}|^2}{|\alpha_{\mathrm{R}}|^2}$.

Combining (1), (2), and (5) yields

$$\begin{aligned} r_k = &\left(\alpha_{\mathrm{T}}\alpha_{\mathrm{R}} h_{k,m} + \beta^{\dagger}_{\mathrm{T}}\beta_{\mathrm{R}} h^{\dagger}_{-k,n}\right) s_k + \\ &\left(\beta_{\mathrm{T}}\alpha_{\mathrm{R}} h_{k,m} + \alpha^{\dagger}_{\mathrm{T}}\beta_{\mathrm{R}} h^{\dagger}_{-k,n}\right) s^{\dagger}_{-k} + \alpha_{\mathrm{R}} w_k + \beta_{\mathrm{R}} w^{\dagger}_{-k}, \end{aligned} \tag{7}$$

where $h_{-k,n}$ denotes the channel of the selected antenna with index n at the mirror subcarrier $-k$.

B. Transmit Antenna Selection with Transmit I/Q Imbalance

As can be seen from (4), the received signal results from a superposition of the transmitted symbols s_k, the interfering transmitted symbols s_{-k}, and noise samples. Hence, the signal-to-interference-and-noise-ratio (SINR) at the k-th subcarrier, i.e., SINR_k can be written as

$$\mathrm{SINR}_k = \frac{E_s\,|\alpha_{\mathrm{T}}|^2 |h_{k,m}|^2}{E_s |\beta_{\mathrm{T}}|^2 |h_{k,m}|^2 + \mathcal{N}_0}. \tag{8}$$

Now, we formulate the problem of antenna selection in presence of transmit I/Q imbalance. Since the SINR in (8) only depends on the channel condition in the k-th subcarrier, the performance metric for the optimization can be the instantaneous capacity of the subcarriers k. Therefore, from (8), the index of the optimum selected antenna can be represented as

$$\begin{aligned} m^* &= \arg\max_{m} \log_2 \left(1 + \frac{E_s\,|\alpha_{\mathrm{T}}|^2 |h_{k,m}|^2}{E_s |\beta_{\mathrm{T}}|^2 |h_{k,m}|^2 + \mathcal{N}_0}\right) \\ &= \arg\max_{m} |h_{k,m}|^2, \end{aligned} \tag{9}$$

where the second equality follows from the fact that $\frac{x}{ax+1}$ is an increasing function of x, for $x \geq 0$ and $a > 0$. Hence, the optimum antenna selection for this case is the selection of the antenna with the best instantaneous channel. Note that the second equality in (9) that contains the I/Q imbalance free channel response is shown for the simplicity in the performance analysis. In practice, for the optimum antenna selection, we can use the criteria in the first equality in which only the effective channel with I/Q imbalance parameters is required.

C. Transmit Antenna Selection with Both Transmit and Receive I/Q Imbalances

Next, we consider the system with both transmit and receive I/Q imbalances. From (7), the after detection received SINR at the k-th subcarrier can be calculated as

$$\mathrm{SINR}_k^{\mathrm{TR}} = \frac{|h_{k,m}|^2 + A_{\mathrm{T}}A_{\mathrm{R}}\frac{|h^{\dagger}_{-k,n}h_{k,m}|^2}{|h_{k,m}|^2} + 2\,\mathrm{Re}\{X_S\}}{A_{\mathrm{T}}\frac{|h^{\dagger}_{-k,n}h_{k,m}|^2}{|h_{-k,n}|^2} + A_{\mathrm{R}}|h_{-k,n}|^2 + 2\,\mathrm{Re}\{X_I\} + \lambda_{\mathrm{TR}}} \geq \frac{|h_{k,m}|^2 + A_{\mathrm{T}}A_{\mathrm{R}}\frac{|h^{\dagger}_{-k,n}h_{k,m}|^2}{|h_{k,m}|^2} + 2\,\mathrm{Re}\{X_S\}}{2A_{\mathrm{T}}\frac{|h^{\dagger}_{-k,n}h_{k,m}|^2}{|h_{-k,n}|^2} + 2A_{\mathrm{R}}|h_{-k,n}|^2 + \lambda_{\mathrm{TR}}} \quad (10)$$

where $\lambda_{\mathrm{TR}} = \frac{(1+A_{\mathrm{R}})}{|\alpha_{\mathrm{T}}|^2}\frac{\mathcal{N}_0}{E_s}$, $X_S \triangleq \frac{\alpha_{\mathrm{T}}^*\alpha_{\mathrm{R}}^*\beta_{\mathrm{T}}^*\beta_{\mathrm{R}}}{|\alpha_{\mathrm{T}}|^2|\alpha_{\mathrm{R}}|^2}\, h^{\dagger}_{-k,n}h_{k,m}$, and $X_I \triangleq \frac{\alpha_{\mathrm{T}}\alpha_{\mathrm{R}}\beta_{\mathrm{T}}\beta_{\mathrm{R}}^*}{|\alpha_{\mathrm{T}}|^2|\alpha_{\mathrm{R}}|^2}\, h^{\dagger}_{-k,n}h_{k,m}$. In the inequality above, the triangle inequality is used. By using Cauchy-Schwarz inequality, i.e., $|h_{k,m}h_{-k,n}| \leq |h_{k,m}|\,|h_{-k,n}|$ and ignoring higher order terms in the nominator of (10), the $\mathrm{SINR}_k^{\mathrm{TR}}$ can be simplified to

$$\mathrm{SINR}_{k,m,n}^{\mathrm{TR}} \approx \frac{|h_{k,m}|^2}{2A_{\mathrm{T}}|h_{k,m}|^2 + 2A_{\mathrm{R}}|h_{-k,n}|^2 + \lambda_{\mathrm{TR}}}. \quad (11)$$

Now, we formulate the problem of antenna selection in the presence of transmit and receive I/Q imbalances. The performance metric for the optimization can be sum-rate capacity of the subcarriers k and $-k$[1]. Therefore, the optimum indices for the selected antennas of subcarriers k and $-k$ is given by

$$(m^*, n^*) = \arg\max_{m,n} \left\{\log_2\left(1 + \mathrm{SINR}_{k,m,n}^{\mathrm{TR}}\right) + \log_2\left(1 + \mathrm{SINR}_{-k,n,m}^{\mathrm{TR}}\right)\right\}. \quad (12)$$

Since we only have limited values for $m \in \{1, 2, \ldots, M\}$ and $n \in \{1, 2, \ldots, M\}$, the brute-force search can be used to obtain the optimum solution. Due to the effect of receive I/Q imbalance, two capacity terms in (12) are not separable. This is in contrast to the antenna selection in (9) in which the index of the optimum selected antenna m only depends on the k-th subcarrier's channel coefficients.

III. Ergodic Capacity

In this section, we derive the close-form expressions for the average sum-rate capacity of the TAS OFDM system with transceiver I/Q imbalance.

A. Ergodic Capacity of TAS with Transmit I/Q Imbalance

The average sum-rate channel capacity or ergodic capacity is given by $\overline{C}_k = \mathbb{E}\{C_k\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operation over channels coefficients. When the system is affected by the transmit I/Q imbalance, from (8) we have

$$\overline{C}_k = \mathbb{E}\left\{\log_2\left(1 + \frac{E_s\,|\alpha_{\mathrm{T}}|^2|h_{k,m^*}|^2}{E_s|\beta_{\mathrm{T}}|^2|h_{k,m^*}|^2 + \mathcal{N}_0}\right)\right\}. \quad (13)$$

From (13), the ergodic capacity can be written as $\overline{C}_k \triangleq \overline{U}_1 - \overline{U}_2$ where

$$\overline{U}_1 = \mathbb{E}\left\{\log_2\left(1 + (1 + A_{\mathrm{T}})\frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,|h_{k,m^*}|^2\right)\right\},$$
$$\overline{U}_2 = \mathbb{E}\left\{\log_2\left(1 + A_{\mathrm{T}}\frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,|h_{k,m^*}|^2\right)\right\}. \quad (14)$$

Using the order statistics, the PDF of the $\gamma_{\mathrm{T}} = \frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,|h_{k,m^*}|^2$ can be calculated as

$$p_{\mathrm{T}}(\gamma) = \frac{M\,e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{T}}}}}{\bar{\gamma}_{\mathrm{T}}}\left(1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{T}}}}\right)^{M-1}. \quad (15)$$

where $\bar{\gamma}_{\mathrm{T}} = \frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,\sigma^2_{h_k}$. Next, we find the closed-form expressions for $\overline{U}_1$ and $\overline{U}_2$. By averaging over γ_{T}, $\overline{U}_2$ can be calculated as

$$\begin{aligned}\overline{U}_2 &= \int_0^\infty \log_2\left(1 + A_{\mathrm{T}}\gamma\right)\,p_{\mathrm{T}}(\gamma)\,d\gamma \\ &= \int_0^\infty \log_2\left(1 + A_{\mathrm{T}}\gamma\right)\,\frac{M\,e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{T}}}}}{\bar{\gamma}_{\mathrm{T}}}\left(1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathrm{T}}}}\right)^{M-1}\,d\gamma \\ &= \sum_{i=0}^{M-1}\frac{(-1)^iM}{\bar{\gamma}_{\mathrm{T}}}\binom{M-1}{i}\int_0^\infty \log_2\left(1 + A_{\mathrm{T}}\gamma\right)e^{-\frac{(i+1)\gamma}{\bar{\gamma}_{\mathrm{T}}}}\,d\gamma \\ &= \sum_{i=0}^{M-1}(-1)^i\binom{M-1}{i}\frac{M\log_2(e)}{i+1}\,e^{\frac{i+1}{A_{\mathrm{T}}\bar{\gamma}_{\mathrm{T}}}}E_1\left(\frac{i+1}{A_{\mathrm{T}}\bar{\gamma}_{\mathrm{T}}}\right),\end{aligned} \quad (16)$$

where $E_1(\cdot)$ is the exponential-integral function of first order. From [15, Eq. (8.359)], the exponential integral can be represented in terms of the complementary incomplete gamma function by $E_1(x) = \Gamma(0, x)$ where $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1}e^{-t}dt$ [15, Eq. (8.350)]. We use [16, Eq. (31)] to find the closed-form solution for the integral in the last equation in (16). Similarly, $\overline{U}_1$ can be obtained as

$$\overline{U}_1 = \sum_{i=0}^{M-1}(-1)^i\binom{M-1}{i}\frac{M\log_2(e)}{i+1} \times e^{\frac{i+1}{(1+A_{\mathrm{T}})\bar{\gamma}_{\mathrm{T}}}}E_1\left(\frac{i+1}{(1+A_{\mathrm{T}})\bar{\gamma}_{\mathrm{T}}}\right). \quad (17)$$

Combining (16) and (17), a closed-form solution for the ergodic capacity is obtained as $\overline{C}_k = \overline{U}_1 - \overline{U}_2$.

1) Upper-Bound: By the fact that $\log_2(1+x)$ is a concave function, we get an upper-bound for the ergodic capacity using Jensen's inequality

$$\overline{C}_k \leq \log_2\left(1 + (1 + A_{\mathrm{T}})\frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,\mathbb{E}\{|h_{k,m^*}|^2\}\right) - \log_2\left(1 + A_{\mathrm{T}}\frac{|\alpha_{\mathrm{T}}|^2E_s}{\mathcal{N}_0}\,\exp\left(\mathbb{E}\left\{\log\left[|h_{k,m^*}|^2\right]\right\}\right)\right), \quad (18)$$

where for the second term in (18) we used the fact that $\log_2(1 + a\,e^x)$ is a convex function with $a > 0$. It can be shown that the expectation value of the maximum of M i.i.d. exponential random variables with mean $\sigma^2_{h_k}$ is given by

$$\mathbb{E}\left\{|h_{k,m^*}|^2\right\} = M\sigma^2_{h_k}\sum_{i=0}^{M-1}\binom{M-1}{i}\frac{(-1)^i}{(i+1)^2} \triangleq \phi_k(M). \quad (19)$$

In addition, the expectation value of the logarithm of the maximum of M i.i.d. exponential random variables with mean $\sigma^2_{h_k}$ is given by

[1]From [14], it is known that the capacity of an OFDM system for a fixed channel realization derives as the sum capacity of the individual subcarriers used for data transmission.

$$\mathbb{E}\left\{\log\left[|h_{k,m^*}|^2\right]\right\} = \int_0^\infty \log(\gamma)\, p_{\max}(\gamma)\, d\gamma$$
$$= \int_0^\infty \log(\gamma)\, \frac{M\, e^{-\frac{\gamma}{\sigma^2_{h_k}}}}{\sigma^2_{h_k}} \left(1 - e^{-\frac{\gamma}{\sigma^2_{h_k}}}\right)^{M-1} d\gamma$$
$$= M \sum_{i=0}^{M-1} \binom{M-1}{i} \frac{(-1)^i}{i+1} \left[\log\left(\frac{\sigma^2_{h_k}}{i+1}\right) - \kappa\right] \triangleq \psi_k(M), \tag{20}$$

where $\kappa \approx 0.577$ is Euler's constant, $p_{\max}(\gamma)$ is similar to (15) by replacing $\bar{\gamma}_{\mathrm{T}}$ with $\sigma^2_{h_k}$, and [15, Eq. (4.331)] is used for solving the integral in the second equation. Hence, a closed-form solution for the expression in (18) is given by

$$\overline{C}_k \leq \log_2\left(1 + (1 + A_{\mathrm{T}}) \frac{|\alpha_{\mathrm{T}}|^2 E_s}{\mathcal{N}_0}\, \phi_k(M)\right) - \log_2\left(1 + A_{\mathrm{T}} \frac{|\alpha_{\mathrm{T}}|^2 E_s}{\mathcal{N}_0}\, \exp\left(\psi_k(M)\right)\right). \tag{21}$$

2) Lower-Bound: A lower-bound on the ergodic capacity in (13) can be calculated using the procedure given above for deriving the upper-bound on ergodic capacity. Thus, by replacing concave and convex functions in (21) and again using Jensen's inequality, it can be shown that

$$\overline{C}_k \geq \log_2\left(1 + (1 + A_{\mathrm{T}}) \frac{|\alpha_{\mathrm{T}}|^2 E_s}{\mathcal{N}_0}\, \exp\left(\psi_k(M)\right)\right) - \log_2\left(1 + A_{\mathrm{T}} \frac{|\alpha_{\mathrm{T}}|^2 E_s}{\mathcal{N}_0}\, \phi_k(M)\right). \tag{22}$$

B. Ergodic Capacity of TAS with the Transmit and Receive I/Q Imbalances

Now, we consider the capacity analysis of the case that the TAS OFDM system exposed to both the transmit and receive I/Q imbalances. Finding the ergodic capacity based on the optimum antenna selection strategy given in (12) is complicated. Thus, in this subsection we consider the suboptimal antenna selection as

$$m_0^* = \arg\max_m |h_{k,m}|^2 \quad \text{and} \quad n_0^* = \arg\max_n |h_{-k,n}|^2. \tag{23}$$

If there is no receive I/Q imbalance, this antenna selection strategy becomes optimal. From (11), the ergodic sum-rate capacity at the k-th subcarrier is given by

$$\overline{C}_k = \mathbb{E}\left\{\log_2\left(1 + \frac{|h_{k,m_0^*}|^2}{2A_{\mathrm{T}}|h_{k,m_0^*}|^2 + 2A_{\mathrm{R}}|h_{-k,n_0^*}|^2 + \lambda_{\mathrm{TR}}}\right)\right\}. \tag{24}$$

Furthermore, the ergodic capacity in (24) can be written as $\overline{C}_k \triangleq \overline{V}_1 - \overline{V}_2$ where

$$\overline{V}_1 = \mathbb{E}\left\{\log_2\left(1 + \frac{1 + 2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}|h_{k,m_0^*}|^2 + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}|h_{-k,n_0^*}|^2\right)\right\}, \tag{25}$$

$$\overline{V}_2 = \mathbb{E}\left\{\log_2\left(1 + \frac{2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}|h_{k,m_0^*}|^2 + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}|h_{-k,n_0^*}|^2\right)\right\}. \tag{26}$$

Proposition *1:* Consider two finite sets of independent random variables $\mathcal{X} = \{X_1, \ldots, X_M\}$ and $\mathcal{Y} = \{Y_1, \ldots, Y_M\}$ with exponential distribution and mean of $\overline{X}_i = \sigma_x^2$, $i = 1, \ldots, M$, and $\overline{Y}_j = \sigma_y^2$, $j = 1, \ldots, M$, respectively. The CDF of

$$Z = \max_{i \in \{1,\ldots,M\}} X_i + \max_{j \in \{1,\ldots,M\}} Y_j$$

can be calculated as

$$\Pr\{Z < \zeta\} = \sum_{i=0}^{M} \sum_{j=0}^{M-1} \binom{M}{i} \binom{M-1}{j} \frac{M(-1)^{i+j}}{\sigma_y^2} \times \left(\frac{i}{\sigma_x^2} - \frac{j+1}{\sigma_y^2}\right)^{-1} \left(e^{-\frac{(j+1)\zeta}{\sigma_y^2}} - e^{-\frac{\zeta i}{\sigma_x^2}}\right), \tag{27}$$

and PDF is given by

$$p_Z(\zeta) = \sum_{i=0}^{M} \sum_{j=0}^{M-1} \binom{M}{i} \binom{M-1}{j} \frac{M(-1)^{i+j}}{\sigma_y^2} \times \left(\frac{i}{\sigma_x^2} - \frac{j+1}{\sigma_y^2}\right)^{-1} \left(\frac{i}{\sigma_x^2} e^{-\frac{\zeta i}{\sigma_x^2}} - \frac{(j+1)}{\sigma_y^2} e^{-\frac{(j+1)\zeta}{\sigma_y^2}}\right). \tag{28}$$

Proof: The proof is given in Appendix I. ■

We define the term in the argument of (25) as

$$\gamma_{\mathrm{TR}} \triangleq \frac{1 + 2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}|h_{k,m_0^*}|^2 + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}|h_{-k,n_0^*}|^2.$$

Using Proposition 1, and by replacing $\sigma_x^2 = \frac{1+2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}\sigma^2_{h_k}$, the PDF of γ_{TR} becomes

$$p_{\mathrm{TR}}(\gamma) = \sum_{i=1}^{M} \varLambda_i e^{-\frac{i\lambda_{\mathrm{TR}}\gamma}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}}} + \sum_{j=0}^{M-1} \varDelta_j e^{-\frac{(j+1)\lambda_{\mathrm{TR}}\gamma}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}}, \tag{29}$$

where

$$\varLambda_i = \sum_{j=0}^{M-1} \binom{M}{i} \binom{M-1}{j} \frac{(-1)^{i+j}\lambda_{\mathrm{TR}}^2 M\, i}{(1 + 2A_{\mathrm{T}})\, 2A_{\mathrm{R}}\, \sigma^2_{h_k}\sigma^2_{h_{-k}}} \times \left(\frac{i\,\lambda_{\mathrm{TR}}}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}} - \frac{(j+1)\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}\right)^{-1}, \tag{30}$$

$$\varDelta_j = \sum_{i=0}^{M} \binom{M}{i} \binom{M-1}{j} \frac{(-1)^{i+j}\lambda_{\mathrm{TR}} M}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}} \times \left(1 - \frac{i}{j+1} \frac{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}}\right)^{-1}. \tag{31}$$

Using the derived $p_{\mathrm{TR}}(\gamma)$, the closed-form solution for $\overline{V}_1$ in (25) is calculated as

$$\begin{aligned}\overline{V}_1 &= \int_0^\infty \log_2(1+\gamma)\, p_{\mathrm{TR}}(\gamma)\, d\gamma \\ &= \sum_{i=1}^{M} \varLambda_i \int_0^\infty \log_2(1+\gamma)\, e^{-\frac{i\lambda_{\mathrm{TR}}\gamma}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}}} d\gamma + \sum_{j=0}^{M-1} \varDelta_j \int_0^\infty \log_2(1+\gamma)\, e^{-\frac{(j+1)\lambda_{\mathrm{TR}}\gamma}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}} d\gamma \\ &= \sum_{i=1}^{M} \varLambda_i \frac{\log_2(e)\,(1+2A_{\mathrm{T}})\sigma^2_{h_k}}{i\,\lambda_{\mathrm{TR}}} \times e^{\frac{i\lambda_{\mathrm{TR}}}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}}} E_1\left(\frac{i\,\lambda_{\mathrm{TR}}}{(1+2A_{\mathrm{T}})\sigma^2_{h_k}}\right) \\ &\quad + \sum_{j=0}^{M-1} \varDelta_j \frac{\log_2(e)\, 2A_{\mathrm{R}}\sigma^2_{h_{-k}}}{(j+1)\,\lambda_{\mathrm{TR}}} \times e^{\frac{(j+1)\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}} E_1\left(\frac{(j+1)\,\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}\right).\end{aligned} \tag{32}$$

Similarly, $\overline{V}_2$ can be obtained as

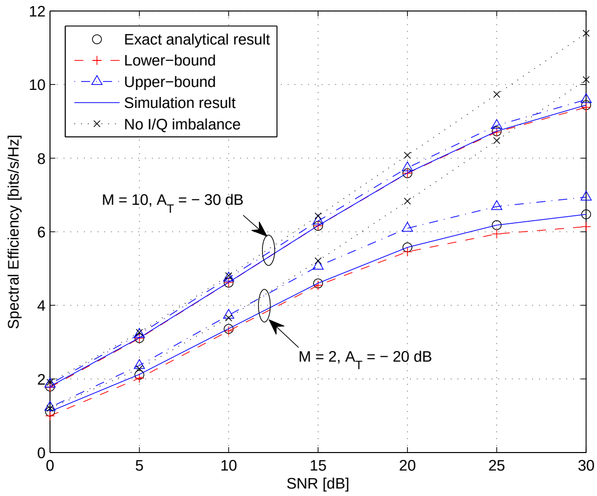

Fig. 1. Ergodic capacity of the k-th subcarrier of TAS OFDM system for different number of transmit antenna M and transmit I/Q imbalances, when $R_k = 1$ bits/s/Hz and $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$, for $k = 1, \ldots, K$.

$$\overline{V}_2 = \sum_{i=1}^{M} \breve{A}_i \frac{\log_2(e)\, 2A_{\mathrm{T}}\sigma^2_{h_k}}{i\,\lambda_{\mathrm{TR}}}\, e^{\frac{i\lambda_{\mathrm{TR}}}{2A_{\mathrm{T}}\sigma^2_{h_k}}} E_1\left(\frac{i\,\lambda_{\mathrm{TR}}}{A_{\mathrm{T}}\sigma^2_{h_k}}\right) + \sum_{j=0}^{M-1} \breve{\Delta}_j \frac{\log_2(e)\, 2A_{\mathrm{R}}\sigma^2_{h_{-k}}}{(j+1)\,\lambda_{\mathrm{TR}}}\, e^{\frac{(j+1)\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}} E_1\left(\frac{(j+1)\,\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}\right), \quad (33)$$

where

$$\breve{A}_i = \sum_{j=0}^{M-1} \binom{M}{i}\binom{M-1}{j} \frac{(-1)^{i+j}\lambda^2_{\mathrm{TR}} M\, i}{2A_{\mathrm{T}}\, 2A_{\mathrm{R}}\, \sigma^2_{h_k}\sigma^2_{h_{-k}}} \times \left(\frac{i\,\lambda_{\mathrm{TR}}}{2A_{\mathrm{T}}\sigma^2_{h_k}} - \frac{(j+1)\lambda_{\mathrm{TR}}}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}\right)^{-1}, \quad (34)$$

$$\breve{\Delta}_j = \sum_{i=0}^{M} \binom{M}{i}\binom{M-1}{j} \frac{(-1)^{i+j}\lambda_{\mathrm{TR}} M}{2A_{\mathrm{R}}\sigma^2_{h_{-k}}} \times \left(1 - \frac{i}{j+1}\frac{2A_{\mathrm{R}}\sigma^2_{h_{-k}}}{2A_{\mathrm{T}}\sigma^2_{h_k}}\right)^{-1}. \quad (35)$$

Combining (32) and (33), the close-form solution for $\overline{C}_k = \overline{V}_1 - \overline{V}_2$ is achieved.

1) Upper-Bound: Define the vector $[x_1, \ldots, x_K]$ of multiple variables. Then, $\log_2(1 + \sum_{k=1}^{K} a_k\, e^{x_k})$ is a convex function on $\mathbb{R}^K$ for arbitrary $a_k > 0$ (see e.g. [17, Lemma 3]). Thus, applying Jensen's inequality in (26), we have

$$\overline{V}_2 \geq \log_2\left(1 + \frac{2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}} \exp\left(\mathbb{E}\left\{\log\left[|h_{k,m_0^*}|^2\right]\right\}\right) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}} \exp\left(\mathbb{E}\left\{\log\left[|h_{-k,n_0^*}|^2\right]\right\}\right)\right). \quad (36)$$

By the fact that $\log_2(1+x)$ is a concave function, we derive an upper-bound for the ergodic capacity at the k-th subcarrier of TAS OFDM system. In order to derive an upper-bound on (24) the above expression, we use Jensen's inequality to obtain

$$\overline{C}_k \leq \log_2\left(1 + \frac{1+2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}\mathbb{E}\left\{|h_{k,m_0^*}|^2\right\} + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}\mathbb{E}\left\{|h_{-k,n_0^*}|^2\right\}\right) - \log_2\left(1 + \frac{2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}} \exp\left(\mathbb{E}\left\{\log\left[|h_{k,m_0^*}|^2\right]\right\}\right) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}} \exp\left(\mathbb{E}\left\{\log\left[|h_{-k,n_0^*}|^2\right]\right\}\right)\right). \quad (37)$$

Using (19) and (20), the close-form solution for the expression above is given as

$$\overline{C}_k \leq \log_2\left(1 + \frac{1+2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}\phi_k(M) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}\phi_{-k}(M)\right) - \log_2\left(1 + \frac{2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}} \exp\left(\psi_k(M)\right) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}} \exp\left(\psi_{-k}(M)\right)\right). \quad (38)$$

2) Lower-Bound: A lower-bound on the ergodic capacity in (14) can be calculated similar to the procedure given above for the upper-band. Thus, we have

$$\overline{C}_k \geq \log_2\left(1 + \frac{1+2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}} \exp\left(\psi_k(M)\right) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}} \exp\left(\psi_{-k}(M)\right)\right) - \log_2\left(1 + \frac{2A_{\mathrm{T}}}{\lambda_{\mathrm{TR}}}\phi_k(M) + \frac{2A_{\mathrm{R}}}{\lambda_{\mathrm{TR}}}\phi_{-k}(M)\right). \quad (39)$$

IV. Numerical Analysis

In this section, numerical results are provided to demonstrate the effectiveness of our analytical results presented in previous sections. In all the evaluation scenarios we have assumed that the channels are frequency selective fading and the magnitude of the frequency domain coefficients $h_{k,m}$ and $h_{-k,n}$ are independent Rayleigh distributed random variables with variance $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$. The transmission rate in each subcarrier R_k is fixed to 1 bits/s/Hz.

In Fig. 1, the ergodic rate of the TAS system at the k-th subcarrier $\overline{C}_k$ in (13) in which affected by transmit I/Q imbalance is depicted. The horizontal axis is transmit SNR, i.e., $\frac{E_s}{N_0}$. As well as the exact capacity expression in terms of the exponential integral, the lower and upper bounds on the

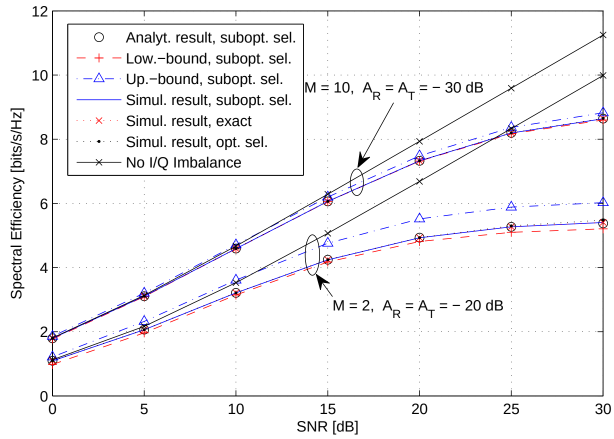

Fig. 2. Performance comparison of analytical and simulated results of ergodic capacity of the k-th subcarrier of TAS OFDM system with both the transmit and receive I/Q imbalances for different number of transmit antenna M and I/Q imbalances, when $R_k = 1$ bits/s/Hz and $\sigma^2_{h_k} = \sigma^2_{h_{-k}} = 1$, for $k = 1, \ldots, K$.

$$\begin{aligned}
\Pr\{Z<\zeta\} &= \int_0^{\infty} \Pr\{X<\zeta-y\}\, p_y(y)\, dy = \int_0^{\zeta} \left(1-e^{-\frac{\zeta-y}{\sigma_x^2}}\right)^M \frac{M\, e^{-\frac{y}{\sigma_y^2}}}{\sigma_y^2} \left(1-e^{-\frac{y}{\sigma_y^2}}\right)^{M-1} dy \\
&= \sum_{i=0}^{M}\sum_{j=0}^{M-1} \binom{M}{i}\binom{M-1}{j} \frac{M(-1)^{i+j} e^{-\frac{\zeta i}{\sigma_x^2}}}{\sigma_y^2} \int_0^{\zeta} e^{y\left(\frac{i}{\sigma_x^2}-\frac{j+1}{\sigma_y^2}\right)} dy \\
&= \sum_{i=0}^{M}\sum_{j=0}^{M-1} \binom{M}{i}\binom{M-1}{j} \frac{M(-1)^{i+j}}{\sigma_y^2} e^{-\frac{\zeta i}{\sigma_x^2}} \left(e^{\zeta\left(\frac{i}{\sigma_x^2}-\frac{j+1}{\sigma_y^2}\right)}-1\right)\left(\frac{i}{\sigma_x^2}-\frac{j+1}{\sigma_y^2}\right)^{-1}. \qquad (40)
\end{aligned}$$

average capacity derived in Subsection III-A are also depicted. We consider I/Q imbalance IRRs are around -30 dB to -20 dB [13], we consider two cases of $A_T = -30, -20$ dB. Fig. 1 confirms that the closed-form expression for the ergodic capacity derived in Subsection III-A has the same performance as simulations. As it can be seen, the upper and lower bounds are tight, especially for the case of $M = 10$ and $A_T = -30$ dB. From the figure, we observe that the lower-bound based on (22) is very close to the capacity. Note that the lower bound on capacity is usually more important for system optimization. As expected, Fig. 1 demonstrates that the effect of transmit I/Q imbalance on the average capacity is more serious in high SNR regime.

Fig. 2 shows the ergodic rate of the TAS system at the k-th subcarrier $\overline{C}_k$ in (13) in which the system suffers from both the transmit and receive I/Q imbalances. The analytical results are based on the approximation in (8), and the suboptimal antenna selection given in (23). The closed-form ergodic capacity in terms of exponential integral is given in Subsection III-B-1. The lower and upper bound curves are based on (39) and (38), respectively. Fig. 2 confirms that the closed-form expression for the ergodic capacity derived in Subsection III-B can approximate the simulation result with a very good precision. Furthermore, we have simulated the ergodic capacity with the optimal antenna selection based on (12), and the exact ergodic capacity which is based on the SINR in (11). One can observe that for both the cases the simulated curves are very close to the capacity of the curve corresponded to the suboptimal antenna selection and the approximated SINR, which shows the effectiveness of our approximations. From (24), and by using the definition of multiplexing gain [18]

$$G_{r,k} = \lim_{\mathrm{SNR}_s \to \infty} \frac{C_k}{\log_2(\mathrm{SNR}_s)}$$

where $\mathrm{SNR}_s = \frac{E_s}{N_0}$, it can be shown that $G_{r,k} = 0$, when $A_T > 0$ or $A_R > 0$. However, when $A_T = 0$ and $A_R = 0$, we have $G_{r,k} = 1$. The saturation of the curves in Fig. 1 and Fig. 2 confirm this fact.

V. Conclusion

In this paper, we studied the effect of I/Q imbalance on TAS OFDM system in frequency selective fading channels with independent Rayleigh distributed coefficients. It was shown that the system with transmit I/Q imbalance has the same capacity maximizing antenna selection as a system with no I/Q imbalance. However, the capacity maximizing antenna selection with receive I/Q depends on the channels of the subcarrier k as well as its mirror subcarrier. We found some exact closed-form expressions for ergodic capacity of the system impaired by either transmit or receive I/Q imbalances. Then, some simple and tight lower and upper bounds formula for the ergodic capacity of the TAS OFDM system with the transmit or receive I/Q imbalances were derived. The numerical results confirmed the correctness of the derived expressions and also show the impact of I/Q imbalances on the capacity of the system. Since we proposed some simple closed-form expressions for average capacity, the system engineer can decide when the effect of IQI imbalance is severe, and thus, IQI compensation methods are employed. Otherwise, the interference due to the IQI can be neglected. Specially, when we operate in high SNR scenario, the impact of IQI should be considered for estimating the physical layer rate.

VI. Acknowledgment

We would like to thank Are Hjørungnes (deceased) from UNIK – University Graduate Center, University of Oslo, Norway, for his helpful suggestions on this work. In May 2011, we were shaken by the tragic loss of our friend and colleague Are Hjørungnes at the age of 40. Are was a dear colleague with a lot of energy, full of life and initiatives and passion for science. Our compassion and thoughts are with his family and friends for their loss.

Appendix I
Proof of Proposition 1

We define $X = \max_{i\in\{1,\dots,M\}} X_i$ and $Y = \max_{j\in\{1,\dots,M\}} Y_j$ which have the distribution like (15) with the parameters σ_x^2 and σ_y^2, respectively. By marginalizing over the random variable Y, the CDF of $Z = X + Y$ can be calculated as (40). Thus, the CDF in (27) is derived. By derivation of CDF over ζ, the PDF in (28) is achieved.

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