

Cooperative Sensing with Joint Energy and Correlation Detection in Cognitive Radio Networks

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Abstract—Cognitive radio networks are a promising solution to address the spectrum shortage issue. Spectrum sensing plays a key role to make such a scenario practical. In this paper, we propose a solution for the problem of spectrum sensing in cognitive radio networks aiming to increase the performance of the cooperative spectrum sensing. We design a cooperative spectrum sensing with detection based on two summary statistics, including energy and first order correlation of the received samples of the signal. The probabilities of detection and false alarm are obtained as criteria for evaluating the performance of the cognitive radio networks. The performance evaluation is presented through simulations. The results show that the proposed method significantly improves the detection performance in cooperative spectrum sensing. It also highly decreases the probability of miss-detection in comparison to the traditional energy detection method.

Index Terms—Cooperative spectrum sensing, cognitive radio networks, energy and correlation detection.

I. INTRODUCTION

Cognitive radio networks (CRNs) have been emerged as a promising technology to mitigate the spectrum shortage problem. The CRNs should be designed for opportunistic access to the unused primary spectrum without causing harmful interference to the primary users (PUs) [1]. Hence, the spectrum sensing plays a key role in identifying this opportunity. Through spectrum sensing, cognitive radio (CR) users can recognize and exploit the unused spectrum channels or holes whenever they are vacated by PUs.

A traditional spectrum sensing technique has been proposed based on energy detection in [2]. Authors in [3] focused on cooperation among multiple secondary users (SUs) which improves the spectrum sensing performance in CR networks. In [4], an efficient linear cooperation framework for spectrum sensing is proposed, where the global decision is a linear combination of the local statistics collected from individual CRs using energy detection.

The covariance information among the received signals (or samples) has been also utilized in the literature. The authors in [5] employed the covariance matrix of the received samples to generate a test statistic by utilizing the fact that the desired signal leads to a larger covariance when PUs are present. In [6], exploiting Stokes sub-vector, two new blind detectors are proposed, namely component correlation based energy-polarization detection and vector correlation based energy-polarization detection. In [7], the authors proposed sensing methods using the repetition structure of the pilots in OFDM-based systems. The derived approximate likelihood ratio test

methods only uses the sample auto-correlations of the received signal. In [8], the authors aim to explore correlation information by over-sampling at the receiver for cognitive radios to enhance the performance of the spectrum sensing. However, none of these works consider the first order correlation of the received signal with energy detection in their schemes.

In this paper, we investigate the problem of cooperative spectrum sensing using correlation-based detection. We aim to reduce the probability of miss-detection since it can cause interference to the primary users. To achieve this goal, we introduce two summary statistics representing the energy of the received signals, and first order correlation of the received samples. These statistics are used for the spectrum sensing detection. We first introduce a novel summary statistic as the first order correlation of the received samples. Then, the spectrum sensing is formulated based on the energy and first order correlation detection. We derive the probability of false alarm and miss-detection for different scenarios of spectrum sensing using linear combination of the aforementioned summary statistics. The use of linear combination results in low computational complexity and closed-form expressions. The linear combination of these statistics can lead to a new statistic in which the distance between the mean of the distributions for both hypotheses tests is higher. This results in a lower variance for any given SNR. Our differentiation with existing works lies in correlation of the samples. In case that the signal samples are correlated, our proposed scheme considerably improves the performance of the sensing. However, in lack of correlation between the samples, the performance of the proposed scheme is the same as traditional energy detection schemes. The simulation results show that the proposed cooperative spectrum sensing decreases the probability of miss-detection which improves the performance of a cognitive radio network.

The rest of this paper is organized as follows. In Section II, we define energy and first order correlation summary statistics. In Section III, different scenarios for the spectrum sensing is considered and the probabilities of detection and false alarm are derived. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

A cognitive radio network consisting of M users is considered. The spectrum sensing aims to determine which one of the two hypotheses \mathcal{H}_0 , i.e., when the PU is not active, and \mathcal{H}_1 , i.e., when the PU is active, happens. Under these two hypotheses at k -th time instance for $k = 1, 2, \dots, N$, the received signal $x_i(t)$ by the i -th SU is given by

$$\begin{cases} \mathcal{H}_0 : x_i(k) = v_i(k), \\ \mathcal{H}_1 : x_i(k) = h_i s(k) + v_i(k), \forall i = 1, 2, \dots, M, \end{cases} \quad (1)$$

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where $v_i(k)$ and $s(k)$ denote the additive white Gaussian noise (AWGN) with $v_i(k) \sim \mathcal{CN}(0, \sigma_i^2)$ for the i -th SU and the transmitted signal from the PU at time k , respectively. Let h_i denote the channel gain between the primary transmitter and the SU i receiver. We assume that the exact channel power gains can be estimated if the secondary users know the primary transmit power. We assume that the transmitted signal $s(k)$ and the set of noises $\{v_i(k)\}$ are independent of each other. Note that usually the samples of transmitted signals are related to the modulation schemes employed such as QAM and FSK, where adjacent samples of the transmitted pulses are correlated with each others. Thus, the sampling rate is chosen in a way that the set of noise samples are independent of each other.

Each SU calculates two statistics u_i and r_i denoting the received energy and the first order correlation for SU i over an interval of N samples, respectively, as follows

$$u_i \triangleq \sum_{k=0}^{N-1} |x_i(k)|^2, \quad i = 1, 2, \dots, M, \quad (2)$$

$$r_i \triangleq \sum_{k=0}^{N-2} x_i(k+1)x_i^*(k), \quad i = 1, 2, \dots, M. \quad (3)$$

Assuming $|x_i(k)|^2$ for $i = 1, \dots, M$ to be independent and identically distributed (i.i.d) random variables, and based on the central limit theorem [9], for large values of N , the statistics $\{u_i\}$ are approximately normally distributed with means

$$\mathbb{E}[u_i] = \begin{cases} N\sigma_i^2 & \mathcal{H}_0, \\ (N + \zeta_i)\sigma_i^2 & \mathcal{H}_1, \end{cases} \quad (4)$$

where $\zeta_i \triangleq \frac{E_s|h_i|^2}{\sigma_i^2}$. We define E_s as

$$E_s \triangleq \sum_{k=0}^{N-1} |s(k)|^2. \quad (5)$$

The variance can be written as

$$\text{Var}[u_i] = \begin{cases} 2N\sigma_i^4 & \mathcal{H}_0, \\ 2(N + 2\zeta_i)\sigma_i^4 & \mathcal{H}_1. \end{cases} \quad (6)$$

The same argument can be suggested for r_i with slight modifications. In this case, the terms $x_i(k+1)x_i^*(k)$ and $x_i(k+2)x_i^*(k+1)$ are not independent of each other. Thus, we consider them as two i.i.d random variable sets. Based on the central limit theorem [9], for sufficiently large values of N , each of these sets approximately follows a normal distribution. As a result, statistics $\{r_i\}$ have normal distribution (approximately) with means

$$\mathbb{E}[r_i] = \begin{cases} 0 & \mathcal{H}_0, \\ |h_i|^2 R_s(1) & \mathcal{H}_1, \end{cases} \quad (7)$$

where

$$R_s(1) \triangleq \sum_{k=0}^{N-2} s_i(k+1)s_i^*(k), \quad (8)$$

and the variance is given by

$$\text{Var}[r_i] \simeq \begin{cases} (N-1)\sigma_i^4 & \mathcal{H}_0, \\ (N-1)\sigma_i^4 + 2E_s|h_i|^2\sigma_i^2 & \mathcal{H}_1. \end{cases} \quad (9)$$

In equation (9), we assume that for large values of N , the term $\sum_{k=0}^{N-2} |s(k+1)|^2 + |s(k)|^2$ can be approximated by $2E_s$. The covariance between u_i and r_i can be written as

$$\text{Cov}[u_i, r_i] = \begin{cases} 0 & \mathcal{H}_0, \\ 2|h_i|^2\sigma_i^2 R_s(1) & \mathcal{H}_1. \end{cases} \quad (10)$$

III. LOCAL AND GLOBAL SPECTRUM SENSING

In this section, we consider local and global scenarios for spectrum sensing.

A. Local Sensing

In this case, we consider the spectrum sensing for each secondary user individually. Each SU i employs a linear combination of u_i and r_i as the test statistic. We define this linear combination as

$$z_i = w_{u,i}u_i + w_{r,i}r_i, \quad (11)$$

where $w_{u,i}$ and $w_{r,i}$ denote the combining weights for statistics u_i and r_i , respectively. These coefficients represent the contribution of each statistic to the decision. Based on (11), and using (4) and (7), the mean of z_i can be written as

$$\mathbb{E}[z_i] = \begin{cases} w_{u,i}N\sigma_i^2 & \mathcal{H}_0, \\ w_{u,i}(N + \zeta_i)\sigma_i^2 + w_{r,i}|h_i|^2 R_s(1) & \mathcal{H}_1, \end{cases} \quad (12)$$

Using (6), (9), and (10), the variance of z_i , under different hypotheses, are written as

$$\text{Var}[z_i|\mathcal{H}_j] = \mathbf{w}_i^T \Sigma_{\mathcal{H}_j} \mathbf{w}_i, \quad j = 0, 1, \quad (13)$$

where $\mathbf{w}_i = (w_{u,i}, w_{r,i})$ and

$$\Sigma_{\mathcal{H}_0} = \begin{bmatrix} 2N\sigma_i^4 & 0 \\ 0 & (N-1)\sigma_i^4 \end{bmatrix}, \quad (14)$$

and

$$\Sigma_{\mathcal{H}_1} = \begin{bmatrix} (N + \zeta_i)\sigma_i^2 & 2|h_i|^2\sigma_i^2 R_s(1) \\ 2|h_i|^2\sigma_i^2 R_s^*(1) & 2E_s|h_i|^2\sigma_i^2 + (N-1)\sigma_i^4 \end{bmatrix}. \quad (15)$$

Typically, the performance of spectrum sensing is evaluated with the probability of detection and probability of false alarm. A missed detection of the PU's can cause interferences to them, while a false alarm causes the SUs to loose an opportunity to use the unoccupied channel. Therefore, the probability of detection can be considered as a measure for the spectrum sensing performance. For local sensing, the decision rule is chosen as

$$z_i \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma_i, \quad (16)$$

where γ_i is the decision threshold for SU i . Based on this, the probability of false alarm, P_F , is formulated as [4]

$$P_F = \mathcal{P}(\mathcal{H}_1|\mathcal{H}_0) = Q\left(\frac{\gamma_i - \mathbb{E}[z_i|\mathcal{H}_0]}{\sqrt{\text{Var}[z_i|\mathcal{H}_0]}}\right), \quad (17)$$

where $\mathcal{P}(\mathcal{H}_1|\mathcal{H}_0)$ is the probability that \mathcal{H}_1 is true, i.e., the PUs are active, while actually \mathcal{H}_0 is true, i.e., the PU is not active. The probability of detection, P_D , can be written as

$$P_D = \mathcal{P}(\mathcal{H}_1|\mathcal{H}_1) = Q\left(\frac{\gamma_i - \mathbb{E}[z_i|\mathcal{H}_1]}{\sqrt{\text{Var}[z_i|\mathcal{H}_1]}}\right), \quad (18)$$

where $\mathcal{P}(\mathcal{H}_1|\mathcal{H}_1)$ is the probability that \mathcal{H}_1 is true, i.e., the PUs are active and actually \mathcal{H}_1 is true, i.e., the PU is active. By some manipulations on (17), γ_i is given by

$$\gamma_i = \sqrt{\text{Var}[z_i|\mathcal{H}_0]}Q^{-1}(P_F) + \mathbb{E}[z_i|\mathcal{H}_0]. \quad (19)$$

Substituting (19) in (18), one can obtain

$$P_D = Q\left(\frac{\sqrt{\mathbf{w}_i^T \Sigma_{\mathcal{H}_0} \mathbf{w}_i} Q^{-1}(P_F) - \mathbf{g}_i^T \mathbf{w}_i}{\sqrt{\mathbf{w}_i^T \Sigma_{\mathcal{H}_1} \mathbf{w}_i}}\right), \quad (20)$$

where $\mathbf{g}_i = (\zeta_i \sigma_i^2, R_s(1)|h_i|^2)$. For a given probability of false alarm, we aim to maximize the probability of detection. The problem of maximizing the probability of detection is equivalent to the following optimization problem

$$\min_{\mathbf{w}_i} f(\mathbf{w}_i), \quad (21)$$

where

$$f(\mathbf{w}_i) = \frac{\sqrt{\mathbf{w}_i^T \Sigma_{\mathcal{H}_0} \mathbf{w}_i} Q^{-1}(P_F) - \mathbf{g}_i^T \mathbf{w}_i}{\sqrt{\mathbf{w}_i^T \Sigma_{\mathcal{H}_1} \mathbf{w}_i}}. \quad (22)$$

We note that the decision threshold directly relates to the weight vector \mathbf{w}_i for a given targeted probability of false alarm P_f . The optimal decision threshold is achieved by solving the optimization problem (21) over weight vector \mathbf{w}_i . This optimization problem can be solved by the proposed methods in [4] or [10].

B. Global Sensing

In this section, both statistics u_i and r_i are transmitted to the fusion center via a noisy control channel and a global test statistic is calculated in fusion center for detection. So the received signals at fusion center can be written as

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_u \\ \mathbf{y}_r \end{bmatrix} = \begin{bmatrix} \mathbf{u} \\ \mathbf{r} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_u \\ \mathbf{n}_r \end{bmatrix}, \quad (23)$$

where $\mathbf{u} = (u_1, u_2, \dots, u_M)$ and $\mathbf{r} = (r_1, r_2, \dots, r_M)$. We define $\mathbf{n}_u = (n_{u_1}, \dots, n_{u_M}) \sim \mathcal{N}(\mathbf{0}, \text{diag}(\boldsymbol{\delta}_u))$ in which the variances are collected into the vector form $\boldsymbol{\delta}_u = (\delta_{u_1}^2, \delta_{u_2}^2, \dots, \delta_{u_M}^2)$, $\mathbf{n}_r = (n_{r_1}, \dots, n_{r_M}) \sim \mathcal{N}(\mathbf{0}, \text{diag}(\boldsymbol{\delta}_r))$ in which the variances are collected into the vector form $\boldsymbol{\delta}_r = (\delta_{r_1}^2, \delta_{r_2}^2, \dots, \delta_{r_M}^2)$. The received signals are normal with means $\mathbb{E}[y_{u_i}] = \mathbb{E}[u_i]$ and $\mathbb{E}[y_{r_i}] = \mathbb{E}[r_i]$ and variances $\text{Var}[y_{u_i}] = \text{Var}[u_i] + \delta_{u_i}^2$ and $\text{Var}[y_{r_i}] = \text{Var}[r_i] + \delta_{r_i}^2$. We define the global test statistic as linear combination of all received signals as follows

$$z_c = \mathbf{w}_u^T \mathbf{y}_u + \mathbf{w}_r^T \mathbf{y}_r = \mathbf{w}_f^T \mathbf{y}, \quad (24)$$

where $\mathbf{w}_f = (\mathbf{w}_u, \mathbf{w}_r)$ in which $\mathbf{w}_u = (w_{u_1}, w_{u_2}, \dots, w_{u_M})$, and $\mathbf{w}_r = (w_{r_1}, w_{r_2}, \dots, w_{r_M})$ are the weight vectors representing the contribution of each received signal in the final decision. The mean of z_c can be formulated as

$$\mathbb{E}[z_c] = \begin{cases} N \mathbf{w}_u^T \boldsymbol{\sigma} & \mathcal{H}_0, \\ \mathbf{w}_u^T (N \boldsymbol{\sigma} + E_s \mathbf{h}) + \mathbf{w}_r^T \mathbf{h} R_s(1) & \mathcal{H}_1, \end{cases} \quad (25)$$

where $\mathbf{h} = (|h_1|^2, |h_2|^2, \dots, |h_M|^2)$, $\boldsymbol{\sigma} = (\sigma_1^2, \sigma_2^2, \dots, \sigma_M^2)$. The variances under different hypotheses are written as

$$\text{Var}[z_c|\mathcal{H}_i] = \mathbf{w}_f^T \tilde{\Sigma}_{\mathcal{H}_i} \mathbf{w}_f, \quad i = 0, 1 \quad (26)$$

where

$$\tilde{\Sigma}_{\mathcal{H}_0} = \begin{bmatrix} 2N \text{diag}^2(\boldsymbol{\sigma}) + \text{diag}(\boldsymbol{\delta}_u) & \mathbf{0}_{M \times M} \\ \mathbf{0}_{M \times M} & (N-1) \text{diag}^2(\boldsymbol{\sigma}) + \text{diag}(\boldsymbol{\delta}_r) \end{bmatrix}, \quad (27)$$

and $\tilde{\Sigma}_{\mathcal{H}_1}$ is defined at the bottom of the page as equation (34). In this case, the detector can be defined as

$$z_c \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma_0, \quad (29)$$

and similar to (20), probability of detection can be written as

$$P_D = Q\left(\frac{\sqrt{\mathbf{w}_f^T \tilde{\Sigma}_{\mathcal{H}_0} \mathbf{w}_f} Q^{-1}(P_F) - \mathbf{g}^T \mathbf{w}_f}{\sqrt{\mathbf{w}_f^T \tilde{\Sigma}_{\mathcal{H}_1} \mathbf{w}_f}}\right), \quad (30)$$

where $\mathbf{g} = (E_s \mathbf{h}, R_s(1) \mathbf{h})$. For a given probability of false alarm, the problem of maximizing the probability of detection is equivalent to the following optimization problem

$$\min_{\mathbf{w}_f} f(\mathbf{w}_f), \quad (31)$$

where

$$f(\mathbf{w}_f) = \frac{\sqrt{\mathbf{w}_f^T \tilde{\Sigma}_{\mathcal{H}_0} \mathbf{w}_f} Q^{-1}(P_F) - \mathbf{g}^T \mathbf{w}_f}{\sqrt{\mathbf{w}_f^T \tilde{\Sigma}_{\mathcal{H}_1} \mathbf{w}_f}}. \quad (32)$$

Similar to (25), the aforementioned optimization problem can be solved using the proposed methods in [4] or [10].

IV. NUMERICAL RESULTS

In this section, the proposed scheme is evaluated numerically for different scenarios. We consider M CR users in our cognitive radio network topology. For simplicity, we assume that the transmitted primary signal is $s(k) = 1$ for all Figs. except Fig. 3. We compared the results of our proposed spectrum sensing with the cooperative energy detection spectrum sensing presented in [4]. Thus the simulation parameters are considered as in [4].

In Fig. 1, in order to setup the simulation, we consider a cognitive radio network with $M = 6, N = 20, \sigma_i^2 = 1, \forall i$. We also considered the local SNRs at each CRs as [9.3, 7.8, 9.6, 7.6, 3.5, 9.2] in dB. The results are achieved based on 10^6 noise realizations. Fig. 2 shows the minimum probability of miss detection ($1 - P_D$) versus the probability of false alarm. Based on this results, it is observed that proposed method based on joint correlation and energy detection outperforms the detection performance for spectrum sensing in comparison to energy detection. For Fig. 2, the

$$\tilde{\Sigma}_{\mathcal{H}_1} = \begin{bmatrix} 2N \text{diag}^2(\boldsymbol{\sigma}) + 4E_s \text{diag}(\boldsymbol{\sigma}) \text{diag}(\mathbf{h}) + \text{diag}(\boldsymbol{\delta}_u) & 2R_s(1) \text{diag}(\boldsymbol{\sigma}) \text{diag}(\mathbf{h}) \\ 2R_s^*(1) \text{diag}(\boldsymbol{\sigma}) \text{diag}(\mathbf{h}) & (N-1) \text{diag}^2(\boldsymbol{\sigma}) + 2E_s \text{diag}(\boldsymbol{\sigma}) \text{diag}(\mathbf{h}) + \text{diag}(\boldsymbol{\delta}_r) \end{bmatrix} \quad (34)$$

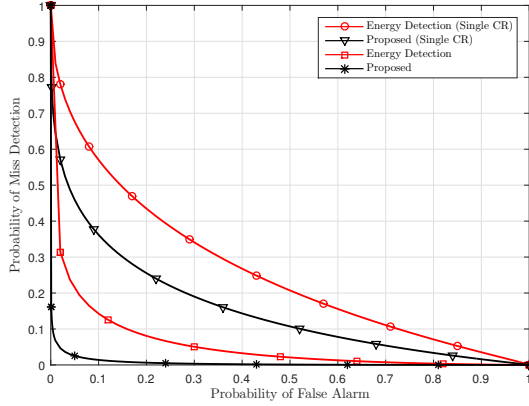


Fig. 1. Probability of miss detection versus false alarm for different scenarios under traditional energy detection and the proposed approach for $M = 6$.

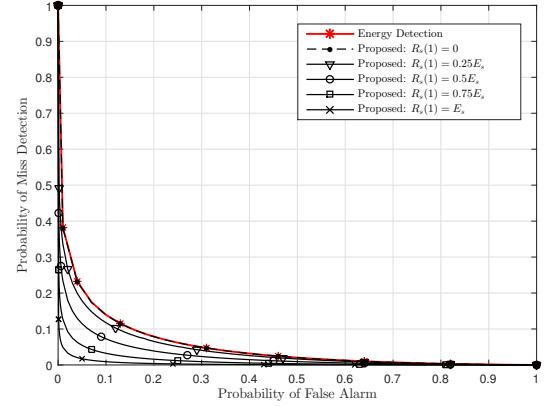


Fig. 3. Effect of correlation between transmitted signal samples on performance of the proposed joint energy and correlation based spectrum sensing.

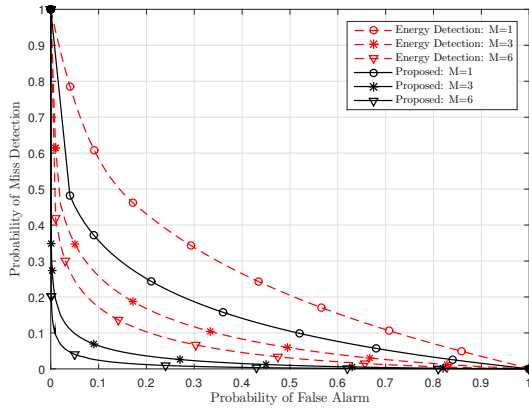


Fig. 2. Probability of miss detection versus probability of false alarm under traditional energy detection and the proposed approach for $M = 1, 3, 6$.

simulation parameters are $N = 20$, $\delta_{ui} = \delta_{ri} = 1, \forall i$ and the curves are depicted for different values of $M = 1, 3$, and 6 , where the corresponding values of average local SNRs are $8.3, 7.5$, and 5.9 in dB. For $M = 1$, the noise level is considered as $\sigma_1^2 = 1.9$ and the local SNR is 8.3 . For $M = 3$, the noise level is $\sigma = \{0.7, 1.0, 0.8\}$ and the local SNRs are $\{10.4, 9.3, 2.6\}$ dB. For $M = 6$, the noise levels are considered as $\sigma = \{0.9, 1.3, 1.0, 2.0, 1.8, 1.2\}$ and local SNRs are $\{7.2, 5.1, 10.8, -1.2, 3.6, 9.7\}$ in dB. Fig. 2 shows that the proposed method significantly decreases the probability of miss detection in comparison to traditional energy detection. Moreover, by increasing the number of cooperative CRs, the performance of spectrum sensing is improved.

In Fig. 3, the simulation parameters are set as the same as those of Fig. 1 except the transmitted signal. In Fig. 3, the effect of the first order correlation between the samples of the transmitted signal is investigated. For this aim, a signal with different values of correlation between its samples is generated in which $R_s(1)$ indicates the extent to which adjacent signal samples are correlated. It is observed that when the adjacent samples of the signal are not correlated, the result of the proposed method is equal to that of the

traditional energy detection method. However, by increasing the correlation between the samples of transmitted signal, the probability of miss detection decreases, resulting better performance for spectrum sensing. Consequently, the proposed method improves the performance of spectrum sensing whenever the adjacent samples of received signals are correlated.

V. CONCLUSION

In this paper, we proposed a novel solution for the problem of cooperative spectrum sensing in cognitive radio networks to decrease the probability of false alarm and miss-detection. We introduced two summary statistics including the traditional energy and first order correlation of the received samples. Then, the spectrum sensing problem was formulated based on them. The proposed scheme has been evaluated through simulations, and the results revealed that proposed scheme has better performance in comparison to the energy detection.

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