Outage Probability Analysis of the Millimeter-Wave Relaying Systems

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Abstract—Millimeter-wave (mm-wave) communication is a promising technology for the next generation wireless networks. Motivated by the immense amount of bandwidth at these bands, it can be used to support the quality of service requirements for the bandwidth-intensive purposes, like the backhaul demands of the small cell base stations. However, the high path loss, the limited penetration ability and the intermittent connectivity necessitate utilizing the multi-hop transmission techniques to maintain network connectivity. In this paper, the outage performance of the mm-wave relaying systems is studied. We take into account the unique propagation characteristics of the mm-wave bands, namely the intermittent connectivity, and obtain the closed-form expression for the outage probability of the mm-wave multi-hop regenerative relaying system. Moreover, the closed-form approximation for the outage probability in a dual-hop nonregenerative case is also derived. The analytical expressions are verified by the simulation results.

I. INTRODUCTION

To accommodate the users’ growing demands of high data rates in wireless communication networks, the system capacity has been improved using advanced signal processing and modulation techniques. Nevertheless, the efficiency of these schemes is restricted because of the narrow bandwidth of the legacy networks. Hence, the millimeter-wave (mm-wave) technology is a promising choice to provide a vast amount of bandwidth for the end users and boost the network capacity. In this respect, there is an increasing effort devoted to the area of the mm-wave communications. However, there are some technical challenges in the development of the cellular communications integrating with the mm-wave technology. The mm-wave signals have a limited ability to penetrate through the obstacles, but they are able to reflect and scatter off of them. Therefore, the signal reception in the outdoor environment greatly depends on the line-of-sight (LOS) links and strong reflected signals resulting from the non-line-of-sight (NLOS) paths [1]. Moreover, Friis law indicates that the omnidirectional path loss increases at higher carrier frequencies. In addition, due to the limited capability of diffraction around the obstacles, the mm-wave signals are more sensitive to blockage in NLOS conditions than the traditional bands. To overcome these difficulties, directional and multi-hop transmission techniques are exploited to bypass the obstacles via relaying and broaden the coverage area. Particularly, due to the inability of the mm-wave signals to penetrate into the indoor environments, relaying is crucial to provide a seamless indoor-outdoor coverage. In addition, by collecting the data traffic of the indoor users who have not link accesses to the mm-wave base station (BS), and transmitting it to the backbone network via relaying, a more uniform quality of service (QoS) throughout the network can be provisioned. Moreover, considering the large number of small cells and depending on their locations, it may be expensive or impractical to provide a fiber connectivity to all cells. Thus, relays are a cost-effective alternative and mm-wave multi-hop communications can be used to provide the insatiable backhaul demands of the small cell BS and mesh-like backhauls. Furthermore, the mm-wave technology also can be employed to support the stringent QoS requirements in the multi-hop device-to-device (D2D) connections [2]. Acute performance analysis of such systems is of paramount importance because it allows to design the network for the maximum utility and performance.

Although the performance of the cooperative systems have been extensively studied, there are only a few studies on the mm-wave cooperative systems [3], [4]. Most of the existing works in the literature have only considered the LOS links for the performance evaluation. However, the NLOS communication is also possible at the mm-wave bands (see, e.g., [5] and [6]). Due to the presence of the several clusters at the receiver in NLOS paths, it is even feasible to support two or three spatial degrees of freedom [1], [7]. In [3], the authors introduce a hop selection metric considering only the LOS links to improve the network flow throughput in the mm-wave wireless personal area networks (WPANs). The authors of [8] presented a distance-based relay selection scheme for the mm-wave WPANs in LOS conditions. In [9], using a stochastic geometry approach the connectivity of the mm-wave relaying systems considering only the LOS links is investigated. The authors of [10] investigate the BS cooperation in the downlink of the mm-wave heterogeneous networks. Also, in a stochastic geometry framework the coverage probability constrained only by blockage is studied. The authors of [11] develop a multi-hop cooperative mm-wave routing protocol for multimedia applications.

In this paper, the outage performance of the mm-wave relaying systems is analyzed. In contrast to the previous works in [8] and [9], in addition to the LOS links, the NLOS paths are also taken into account and the impact of a few received clusters in this condition is investigated. Moreover, we consider the intermittent connectivity problem due to the mm-wave propagation characteristics and it is modeled by a
 LOS probability function. The LOS probability function can be a decreasing function of the distance between the two communicating nodes. Furthermore, the closed-form expressions are derived for the outage probability of the single-hop and multi-hop decode-and-forward (DF) and dual-hop amplify-and-forward (AF) relaying systems operating at the mm-wave frequency bands.

II. SYSTEM AND CHANNEL MODELS

Consider a communication system operating at the mm-wave frequency bands. All nodes are assumed to be equipped with the steerable directional antennas and the mm-wave transceivers. The source node, $R_0$, communicates with the destination node, $R_N$, through $N-1$ intermediate relay nodes. The source node transmits its data signal to the first relay terminal, $R_1$. Intermediate nodes relay the signal from one hop to the next. Note that in the mm-wave frequency bands, directionality is required at both the transmitter and the receiver sides to meet the QoS requirements, such as the minimum received power or the minimum guaranteed data rate. For such networks, network protocols are based on a directional mode operation (with the ability to choose a direction either via beamsteering or sectorization) [4]. In the case that the relays are used to provide the backhaul connectivity, a reliable end-to-end connection is required. Therefore, the intermediate relay terminals should be able to fully decode the received signal and eliminate the noise power, then forward it to the next node. Note that in the regenerative systems the received signal power must meet the QoS requirements to be decoded correctly, otherwise the relay terminal drops the signal to save energy. However, in multi-hop D2D communications usually a smaller number of hops is required to establish a connection between the source and destination devices. Furthermore, it should be noted that both AF and DF schemes require a varying degree of implementation complexity and cost at the mm-wave bands. Hence, due to the space and cost limitations and more importantly limited battery power in the mobile devices, it is more likely to use AF scheme when the devices serve as relays. Thus, in D2D communications where a small number of hops is needed, the mobile devices form a nonregenerative system. In the nonregenerative relaying systems, the $n^\text{th}$ relay node simply amplifies the received signal with the amplification factor $A_n$, and then transmits the amplified signal over the next hop. The amplification factor for the $n^\text{th}$ relay node is given by [12]

$$A_n = \sqrt{\frac{P_n}{P_{n-1}|h_n|^2 + N_{0,n}}}$$

where $P_n$, $n = 1, 2, \ldots, N-1$, is the transmit power of the $n^\text{th}$ node, and $h_n$ and $N_{0,n}$ are respectively the channel gain and the noise power spectral density over the $n^\text{th}$ link between the nodes $R_{n-1}$ and $R_n$. We also assume that all channel links are mutually independent. Under this relay gain assumption, it is shown in [13] that the end-to-end signal-to-noise-ratio (SNR) can be expressed as

$$\gamma_{eq} = \left[ \prod_{n=1}^{N} (1 + \frac{1}{\gamma_n}) - 1 \right]^{-1}$$

where $\gamma_n$ represents the instantaneous SNR of the $n^\text{th}$ hop. The performance analysis of multihop relaying systems using (2) is not tractable and the approximated end-to-end SNR, $\gamma_{eq}$, is widely used in the literature

$$\gamma_{eq} = \frac{1}{\sum_{n=1}^{N} \frac{1}{\gamma_n}}$$

Additionally, Due to the inherent propagation characteristics in the mm-wave bands, we use two different channel models for LOS and NLOS links. The mm-wave channel model for NLOS links is derived by the recent measurements [1] and [7]. Since the mm-wave signals have a limited ability to diffract around obstacles, only a few number of clusters can be detected in the receiver in NLOS conditions. The number of received clusters is modeled with a Poisson-max distribution and the channel model can be stated as [7]

$$h = \sum_{k=1}^{K} g_k, \quad K \sim \max(1, \text{Poi}(\lambda)), \quad g_k \sim CN(0, 2\sigma^2)$$

where $K$ is the number of detected clusters and $g_k$ is the complex-Gaussian fading gain of the $k^\text{th}$ cluster. Note that the parameter $\sigma$, which determines the received power, and also other NLOS link parameters highly depend on the scattering environment. For example, a path loss exponent as large as 5.76 was found in downtown New York City, while only 3.86 was found on the UT Austin campus [1]. However, the authors in [1] and [5] believe that the received power can be large enough to communicate in NLOS conditions. For modeling the LOS links, we consider the Rician channel gain model with the parameters $S^2$ and $2\sigma^2$. Since mm-wave channels suffer from the intermittent connectivity due to the mm-wave propagation characteristics, this phenomenon is modeled by a LOS probability function, which determines the probability that a link experiences LOS situations.

III. OUTAGE PROBABILITY ANALYSIS

A. Single-Hop Outage Probability

The outage probability, $P_{out}$, is defined as the probability that the instantaneous SNR falls below a certain and predetermined threshold SNR, $\gamma_{th}$. Hence, it can be written as

$$P_{out} = P_r(\gamma < \gamma_{th}) = P_r(\gamma < \gamma_{th}|\text{LOS})P_{LOS} + P_r(\gamma < \gamma_{th}|\text{NLOS})(1 - P_{LOS}),$$

where $P_{LOS}$ is the LOS probability function. Our analysis is valid for different $P_{LOS}$, for instance, in [6] a distance-dependent non-increasing LOS probability function is introduced. Next, we try to calculate the conditional probabilities
in (5) as
\[ P_{\text{out},L} = \Pr \{ \gamma < \gamma_{th} | \text{LOS} \} = \Pr \left\{ \frac{|h_{\text{LOS}}|^2}{N_0} < \gamma_{th} \right\} \]
\[ = \Pr \left\{ |h_{\text{LOS}}| < \sqrt{\frac{\gamma_{th} N_0}{P}} \right\} = 1 - Q_1 \left( \frac{\sqrt{\gamma_{th} N_0}}{\sigma_r} \right) \], (6)
where \( Q_1 \) is the marcum Q-function and \( T = \sqrt{\gamma_{th} N_0/P} \). Moreover, using the marginal probability and chain rules, for NLOS links we have
\[ P_{\text{out},N} = \Pr \{ \gamma < \gamma_{th} | \text{NLOS} \} \]
\[ = \Pr \left\{ \frac{|h_{\text{NLOS}}|^2}{N_0} < \gamma_{th} \right\} \]
\[ = \Pr \left\{ \left| \sum_{k=1}^{K} g_k \right| < \sqrt{\frac{\gamma_{th} N_0}{P}} \right\} \]
\[ = \sum_{m=1}^{\infty} \Pr \left\{ \left| \sum_{k=1}^{K} g_k \right| < T \middle| K = m \right\} \Pr \{ K = m \} \]
\[ = e^{-\lambda} \left\{ \left( 1 - e^{-\frac{T^2}{2\sigma^2}} \right) + \sum_{m=1}^{\infty} \left( 1 - e^{-\frac{T^2}{2m\sigma^2}} \right) \left( \frac{\lambda^m}{m!} \right) \right\}. \] (7)

Therefore, combining (6) and (7), the outage probability of a single-hop mm-wave transmission, \( P_{\text{out},s} \), can be stated as
\[ P_{\text{out},s} = \left\{ 1 - Q_1 \left( \frac{s \cdot \frac{T}{\sigma_r}}{\sigma_r} \right) \right\} P_{\text{LOS}} + e^{-\lambda} \left\{ \left( 1 - e^{-\frac{T^2}{2\sigma^2}} \right) + \sum_{m=1}^{\infty} \left( 1 - e^{-\frac{T^2}{2m\sigma^2}} \right) \left( \frac{\lambda^m}{m!} \right) \right\} \left( 1 - P_{\text{LOS}} \right). \] (8)

It is clear from (8) that the outage probability significantly depends on the \( P_{\text{LOS}} \). In a dense urban environment with low \( P_{\text{LOS}} \), the signal reception relies more on the received power from the NLOS paths. In addition, since the NLOS channel parameters highly depend on the scattering environment, the outage probability in NLOS conditions can vary differently depending on the scattering environment and their ability to reflect the mm-wave signals.

**B. Multi-Hop Decode-and-Forward Outage Probability**

In DF relaying systems there is a QoS requirement on the system’s bit rate, thereby the intermediate terminals relay the received signal, if and only if the instantaneous received SNR is above the threshold SNR. Otherwise, the relay drops the received signal to save energy. Therefore, an outage occurs if either one of the hops is in outage, and the overall system outage is dominated by the weakest link.

We assume the uniform power allocation policy that is the total available power of the system is evenly distributed among all the intermediate relay terminals. The nodes are considered to be located at equal distances from each other, hence \( P_{\text{LOS}} \), which is a function of the distance, is equal for all hops. It is assumed that the noise power spectral density at all nodes are the same. Thus, the received SNR at all nodes are independent and identically distributed and the outage probability can be written as
\[ P_{\text{out}} = \Pr \{ \min(\gamma) < \gamma_{th} \} = 1 - \Pr \{ \min(\gamma) > \gamma_{th} \} \]
\[ = 1 - \Pr \{ \gamma_1 > \gamma_{th} \} \Pr \{ \gamma_2 > \gamma_{th} \} \ldots \Pr \{ \gamma_N > \gamma_{th} \} \]
\[ = 1 - (1 - P_{\text{out},s})^N. \] (9)

Since DF relaying requires the nodes to decode the received signal resulting in latency, high computational and implementation complexity and cost at the mm-wave bands, next we study the performance of a mm-wave AF relaying system.

**C. Dual-Hop Amplify-and-Forward Outage probability in NLOS conditions**

AF scheme is more suitable than DF scheme to be adopted for D2D relaying systems. Away from the complexity involved in decoding mm-wave signals, mobile devices are battery-limited. Hence, it is more convenient to simply amplify the received signal rather than spending energy for the decoding process and then amplifying the signal. In this section, we derive a closed-form expression for the outage probability of a dual-hop AF relaying system employing mm-wave technology in NLOS conditions. Thus, using (3), the end-to-end outage probability is given by
\[ P_{\text{out}} = \Pr \{ \gamma_{eq} < \gamma_{th} \} = \Pr \left\{ \frac{1}{\gamma_1 + \gamma_2} < \gamma_{th} \right\} \]
\[ = \Pr \left\{ \frac{N_0}{\sum_{i=1}^{N} |h_i|^2} + \frac{N_0}{\sum_{j=1}^{N} |h_j|^2} < \gamma_{th} \right\}. \] (10)

Equal noise power at each node is considered. Therefore, the outage probability can be expressed as
\[ P_{\text{out}} = \Pr \left\{ \frac{1}{|h_1|^2 + \frac{P_0}{P_1|h_2|^2}} < T^2 \right\} \]
\[ = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Pr \left\{ \frac{1}{G_1 + \frac{AG_2}{1}} < T^2 \middle| K_1 = m, K_2 = n \right\} \times \Pr \{ K_1 = m, K_2 = n \}, \] (11)
where \( G_1 = \left| \sum_{i=1}^{m} g_{i1} \right|^2 \), and \( G_2 = \left| \sum_{j=1}^{n} g_{j2} \right|^2 \) are respectively the channel gains of the first and the second hops and \( A = P_1/P_0 \), is the ratio of the transmit powers. It can be shown that \( G_1 \) and \( G_2 \) are exponentially distributed random variables with the parameters \( \frac{1}{2m\sigma^2} \) and \( \frac{1}{2n\sigma^2} \), respectively. Hence, using the cumulative distribution function of the harmonic mean of two
outage probability in [14], we can write
\[
\Pr \left\{ \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} < T^2 \right\} = \\
\left( 1 - 4T^2\sigma^2/\Delta \right) K_1 \left( 4T^2\sigma^2/\Delta \right) \\
\times \exp \left[ -T^2 \left( \frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_2^2} \right) \right]
\triangleq F(m\sigma^2, A\sigma^2, T^2),
\]
(12)
where \( K_1(.) \) is the first order modified Bessel function of the second kind. Thus, combining (11) and (12), the closed-form expression for the outage probability can be obtained as
\[
P_{\text{out}} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F(m\sigma^2, A\sigma^2, T^2) \Pr\{K_1 = m\} \Pr\{K_2 = n\}
\]
\[
\quad = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F(m\sigma^2, A\sigma^2, T^2) e^{-2\lambda \frac{m+n}{m!}} \\
\quad + \sum_{m=1}^{\infty} \left( F(m\sigma^2, A\sigma^2, T^2) + F(\sigma^2, mA\sigma^2, T^2) \right) e^{-2\lambda \frac{m}{m!}} \\
\quad + F(\sigma^2, A\sigma^2, T^2) e^{-2\lambda}.
\]
(13)
Note that although the expressions in (8) and (13) include infinite summations, it is found that the summands decay very fast with the increase of \( m \) and \( n \), because the value of the parameter \( \lambda \) is around two [7], and also Stirling’s approximation declares that \( m! \) grows as \( \exp(m \ln m) \). Thus, a truncated summation with a finite number of terms will reliably attain the required accuracy. This is because the fact that in the NLOS conditions a higher number of clusters are received with a lower probability.

IV. NUMERICAL RESULTS
In this section, we present some numerical examples to illustrate our results. We also verify the obtained analytical expressions by simulations. The parameter of the Poisson-max distribution, defined in (4), is considered as \( \lambda = 1.8 \), similar to [7]. We set \( P_0/N_0 = 10 \text{ dB} \) and assume the LOS and NLOS channel parameters of \( K_\text{r} = S^2/2\sigma_r^2 = 5 \), and \( \sigma_r^2 = 1 \). In addition, the infinite summations of (8) and (13) were truncated at the 10-th term.

Fig.1 depicts the outage probability as a function of the threshold SNR, for an \( N \)-hop DF relaying system operating at the mm-wave frequency bands, for \( N = 1, 2, 3 \). All terminals are assumed to be located at equal distances from each other. Therefore, using a distance-dependent LOS probability function, all links have the same \( P_{\text{LOS}} \). This figure indicates that the closed-form analytical expression in (8) and (9) are consistent with the simulation results. We also observe that the LOS probability has a prominent impact on the outage probability. Hence, for the applications such as mesh-like backhauls that need a reliable low outage communications, the involving BSs should have LOS link accesses to each other. Otherwise, if the existence of LOS links for small BSs is not guaranteed, the number of implemented relay terminals should be increased to ensure a reliable end-to-end connection by bypassing the obstacles via relaying. Note that the microwave links will not be able to carry the huge traffic loads of backhauls in future wireless networks and fiber connectivity is costly and may be unavailable at some locations. Moreover, benefiting from the LOS transmissions, the transmit power is directed towards the desired destination. Hence, the interference level in mm-wave communication is much lower than the traditional microwave links and this property makes the mm-wave suitable to provide the backhaul demands of dense small cell deployment.

In Fig.2, the performance of a dual-hop mm-wave AF relaying system in NLOS conditions and under different channel parameters is studied. This figure indicates that the analytical expressions in (13) corroborate the simulation results. It is shown that in NLOS conditions the outage probability after two hops is less than 0.1, for \( \gamma_{th} = 6 \text{ dB} \), and becomes worse by increasing the number of the hops. Thereby, mm-wave multi-hop communication in NLOS conditions should not be used for the applications with high reliability QoS requirements, especially when more than two hops are required to transmit the source data signal to the destination node. Thus, in the multi-hop D2D connections if the required number of hops are more than two hops, one of the intermediate nodes should be a preplanned relay terminal with LOS link accesses which is able to fully decode the signal and eliminates the noise power, then forwards it on the next hop.

Fig. 3 illustrates the outage probability versus the number of hops, for \( P_{\text{LOS}} = 0.1, 0.8, 1 \), and \( \gamma_{th} = 5 \text{ dB} \). It is shown that for a mm-wave DF relaying system in an environment with a moderate LOS probability, for example \( P_{\text{LOS}} = 0.8 \), and the threshold SNR of 5 dB, the outage probability is small enough for most of the applications like video transmission.
services, which is a typical scenario regarding the immense amount of bandwidth at the mm-wave band. However, as the LOS probability decreases the performance of the system significantly degrades.

V. CONCLUSION

In this paper, we studied the performance of the AF dual-hop and DF multi-hop relaying systems operating at the mm-wave frequency bands. The closed-form expressions for the outage probability were derived. It was shown by numerical examples how the results in this paper can be used to obtain the performance of the relaying systems on the mm-wave frequency bands. It was also observed that the LOS probability has a prominent impact on the outage probability and mm-wave communication should not be used for the applications with high reliability QoS requirements in the low LOS probability conditions. Furthermore, since the propagation characteristics in NLOS conditions greatly depends on the scattering environment, the outage performance in this condition also can vary in different environments.

REFERENCES


