

Supply-Demand Function Equilibrium for Double Sided Bandwidth-Auction Games

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Abstract—In this paper, we consider a cellular based primary network co-existing with a secondary network. The primary network consists of multiple service providers (SP) and there are several independent users in the secondary network. The SP are operating on the different frequency spectrum and a group of secondary users intend to share these spectrum with the primary services. This situation is formulated as a bandwidth auction game where each user bids a demand curve and each SP offers a supply curve. We consider two cases of complete information case and incomplete information case or learning games. For two cases, we derive the optimal strategies of the players and the distributed algorithms are presented to obtain the solution of these dynamic games.

I. INTRODUCTION

Cognitive radio is a promising method to solve the spectrum scarcity problem [1]–[5]. Certain radio resources could be employed by cognitive radio network, i. e. the secondary system, provided that it does not cause an adverse interference to the primary system, a.k.a. spectrum owner or licensee. In this paper, we focus on a dynamic exclusive-use model [6], in which only one user can exclusively access the spectrum at any particular point in time. However, at different points in time, the users are accessing the spectrum, and the types of wireless service using that spectrum can be changed. In a dynamic exclusive-use model, a spectrum owner can trade its owned spectrum to a cognitive radio user, and thus can earn revenue. This spectrum can then be accessed by a cognitive radio user for a certain period of time (i.e. an on-demand time-bound spectrum lease [7]). This trading is referred to as a secondary market [8]–[11].

In [12], the problem of competitive pricing in a dynamic spectrum access is addressed using an oligopoly market where few firms compete with each other in terms of price to gain the highest profit. For this purpose, a Cournot model is proposed in [12] to model the interaction among secondary users and service providers (SP). However, in Cournot model, it was assumed that all requested bandwidth is provided by the primary user and price is forced from the primary user to secondary users. Authors in [13] proposes a double auction mechanism in

dynamic spectrum sharing problem, which allows the suppliers and the bidders to play an auction game that involves supply strategy for suppliers and price bid strategy for bidders. In this way, suppliers do not have any contribution to the price and bidders do not have any contribution to the amount of bandwidth which is not an appropriate assumption. In addition, non of these works calculated the Nash equilibrium analytically.

In our model, the SPs and users are trying to maximize their own payoff in a bandwidth auction game. It is assumed that each user bids a demand curve and each service provider offers a supply curve which is the price as a function of bandwidth. Each point on the supply and demand functions represents the desired amount of selling and buying bandwidth at the specified level of price for users and service providers, respectively. The clearing price and bandwidth for users and SPs are obtained by balancing the supplies and demands. Then, the problem is solved with complete information and the corresponding Nash equilibrium point of the game is achieved. Next, in incomplete information case, a gradient based method is proposed for learning the optimal decision of the players when the game is repeated. In addition, the convergence of the learning decisions to Nash equilibrium point (which is calculated in complete information case) of the game is examined via the simulation results.

II. SYSTEM MODEL

We consider a cellular based primary network co-existing with a secondary network. The primary network consists of M SPs and there are N users in the secondary network. The SP are operating on the different frequency spectrum F_j , $j = 1, \dots, M$ and a group of secondary users willing to share these spectrum with the primary services. In this case, primary service j wants to sell portions of the available spectrum F_j which is denoted by s_j (e.g., time slots in a time-division multiple access (TDMA)- based wireless access system) at price α_j^S (per unit spectrum or bandwidth) to the secondary users. The secondary users utilize adaptive modulation for transmissions on the allocated spectrum in a time-slotted manner. The spectrum demand of the secondary users depends on the transmission rate due to the adaptive modulation in the

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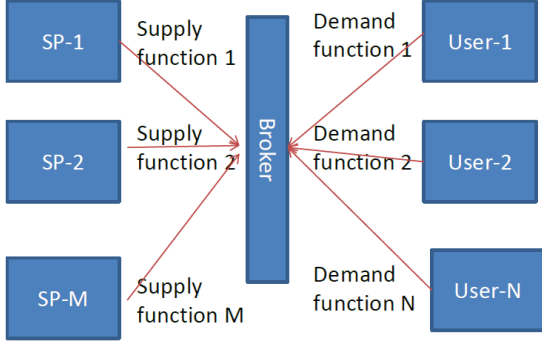


Fig. 1. The bandwidth auction scenario.

allocated frequency spectrum and the price charged by the primary services.

With adaptive modulation, the transmission rate can be dynamically adjusted based on the channel quality. The spectral efficiency of transmission by the i th user can be obtained from [14]

$$k_i = \log_2(1 + K \gamma_i), \quad \text{where } K = \frac{1.5}{\ln(0.2/\text{BER}_{\text{tar}})} \quad (1)$$

where γ_i is the SNR at the receiver, which indicates channel quality, and BER_{tar} is the target bit-error-rate (BER).

In our model, the users and SPs are the players who want to maximize their own payoff in a bandwidth auction game. It is assumed that each user bids a demand curve and each service provider offers a supply curve which is the price as a function of bandwidth. Each point on the supply and demand functions represents the desired amount of selling and buying bandwidth at the specified level of price for users and service providers, respectively.

All the auction activities are conducted by a central entity, called the broker. The broker knows the supply function offered by each SP, and the demand function of every user. Balancing the bandwidth supplies and demands, a uniform price is cleared for all SPs and users. After clearing the price, the broker is responsible for allocating the resources effectively by assigning the bandwidth from SPs to users. This scenario is shown in Fig. 1

III. BANDWIDTH AUCTION GAME MODEL

Considering N competing users and M competing service providers, the demand and supply functions are as follows

$$\begin{aligned} \lambda &= -\alpha_i^D d_i + \beta_i^D, \quad i = 1, 2, \dots, N, \\ \lambda &= \alpha_j^S s_j + \beta_j^S, \quad j = 1, 2, \dots, M, \end{aligned} \quad (2)$$

where λ is the clearing price, d_i is the demand bandwidth variable to user i and s_j is the supply (sold) bandwidth variable of service provider j . The parameters α_i^D , β_i^D represent demand-side bandwidth dependent and demand-side bandwidth-independent prices at the i th user, respectively. Also, α_j^S , and β_j^S represent supply-side bandwidth dependent and supply-side bandwidth-independent prices at the j th SP, respectively. Note that α_i^D , β_i^D , α_j^S , and β_j^S are the positive

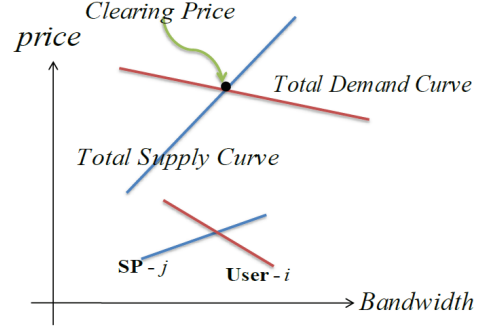


Fig. 2. The bidding supply/demand curves of SP j and user i , respectively. The clearing price is at the crossing point of the equivalent total demand and supply curves.

numbers and represent the strategy of the players. The decreasing demand curves presented in (2) mean that the users buy more in lower prices and less in higher prices and suppliers sell more in higher prices and sell less in lower prices, which is economically meaningful.

The price is cleared when the total supply becomes equal to total demand

$$\sum_{i=1}^N d_i = \sum_{j=1}^M s_j. \quad (3)$$

Fig. 2 shows the bidding supply/demand curves of SP j and user i , respectively. It can be seen that the clearing price is at the crossing point of the equivalent total demand and supply curves.

Using equations (1) and (3) we have

$$\sum_{i=1}^N \frac{\beta_i^D - \lambda}{\alpha_i^D} = \sum_{j=1}^M \frac{\lambda - \beta_j^S}{\alpha_j^S}. \quad (4)$$

Therefore, the price is cleared as follows:

$$\lambda = \frac{\sum_{i=1}^N \frac{\beta_i^D}{\alpha_i^D} + \sum_{j=1}^M \frac{\beta_j^S}{\alpha_j^S}}{\sum_{i=1}^N \frac{1}{\alpha_i^D} + \sum_{j=1}^M \frac{1}{\alpha_j^S}}. \quad (5)$$

Since the price is cleared, the amount of bandwidth for each seller and buyer is determined according to their supply and demand functions as follow

$$\begin{aligned} s_j &= \frac{\lambda - \beta_j^S}{\alpha_j^S}, \quad j = 1, 2, \dots, M, \\ d_i &= \frac{\beta_i^D - \lambda}{\alpha_i^D}, \quad i = 1, 2, \dots, N, \end{aligned} \quad (6)$$

respectively. Also, the following conditions should be held

$$\begin{aligned} 0 &\leq s_j \leq s_j^{\text{Max}}, \quad j = 1, 2, \dots, M, \\ 0 &\leq d_i \leq d_i^{\text{Max}}, \quad i = 1, 2, \dots, N, \end{aligned} \quad (7)$$

where s_j^{Max} and d_i^{Max} are the maximum financial capacity of service provider j and maximum request of user i , respectively. If any of the lower bound inequalities in (7) is

violated, the corresponding demand/supply is set to zero. In the same way, if any of the upper bound inequalities is violated the corresponding demand/supply is set to the maximum capacity/request accordingly. The broker is responsible for purchasing the extra bandwidth from service providers and compensating the lack of supply for users.

The payoff function of service providers is defined as follow

$$\pi_j^S = \lambda s_j - C(s_j) \quad , \quad j = 1, 2, \dots, M, \quad (8)$$

where $C(s_j)$ is the cost of providing the bandwidth equal to s_j which can be considered a linear function of bandwidth as

$$C(s_j) = a_j s_j + b_j \quad (9)$$

where a_j and b_j are some positive constants. The payoff function of each user is also defined as follows

$$\pi_i^D = U(d_i) - \lambda d_i \quad , \quad i = 1, 2, \dots, N \quad (10)$$

where $U(d_i)$ is the utility function of user i , defined as follows [15]

$$U(d_i) = r_i k_i d_i \quad (11)$$

where r_i is the revenue of user i per unit of achievable transmission rate.

The goal of each player, including users and service providers is to adjust his/her strategy (the coefficients of the supply and demand functions) to gain more payoff. However, since the price is a function of the strategies of all players, as it is stated in (5), this problem is a game problem rather than an optimization problem.

The optimal strategy of the players and solution of the game depends on the information available to the players. In the complete information case, the Nash equilibrium point of the game would be an optimal solution of the game, when the game is fully competitive and players decide simultaneously. However, in incomplete information case, the players are not aware of other players' payoff function, and as a result, they cannot reach to equilibrium point in one shot. In the incomplete information case, the players learn their optimal strategy using the available information and historical data gained by the game repetition.

In the following, the problem is solved with complete information and the corresponding Nash equilibrium point of the game is achieved. Next, in incomplete information case, a gradient based method is proposed for learning the optimal decision of the players when the game is repeated. In addition, the convergence of the learning decisions to Nash equilibrium point (which is calculated in complete information case) of the game is examined.

For brevity in the mathematical calculations, in the rest of the paper we assume that α_i^D and α_j^S are some positive fixed constants and β_j^S and β_i^D are the strategies of the service provider j and user i , respectively.

A. Complete information case: Nash equilibrium solution

In the Nash equilibrium point of the game, no player could earn more payoff by individually changing his/her strategy. Considering the reaction function of a player as the best strategy of a player to strategies of other players, Nash equilibrium point can be interpreted as the crossing point of reaction function of all players. The reaction function of a player could be obtained by the first derivative of the payoff function. In our problem, the reaction function of the users and service providers are as follows

$$\frac{\partial \pi_j^S}{\partial s_j} = 0, \quad \frac{\partial \pi_i^D}{\partial d_i} = 0, \quad i = 1, 2, \dots, N, \quad j = 1, 2, \dots, M, \quad (12)$$

Or equivalently, we have

$$\begin{aligned} \frac{\partial \lambda}{\partial \beta_j^S} s_j + \lambda \frac{\partial s_j}{\partial \beta_j^S} - \frac{\partial C(s_j)}{\partial \beta_j^S} &= 0, \\ \frac{\partial U(d_i)}{\partial \beta_i^D} - \lambda \frac{\partial d_i}{\partial \beta_i^D} - \frac{\partial \lambda}{\partial \beta_i^D} d_i &= 0. \end{aligned} \quad (13)$$

After making derivatives, we have

$$\frac{\left(\sum_{i=1}^N \frac{\beta_i^D}{\alpha_i^D} + \sum_{j=1}^M \frac{\beta_j^S}{\alpha_j^S} \right) \left(\frac{2}{K} - \alpha_j^S \right) - a_j (1 - K \alpha_j^S) - \beta_j^S}{K (\alpha_j^S)^2} = 0 \quad (14)$$

$$\frac{\left(\sum_{j=1}^M \frac{\beta_j^S}{\alpha_j^S} + \sum_{j=1}^N \frac{\beta_j^D}{\alpha_j^D} \right) \left(-\alpha_i^D + \frac{2}{K} \right) + r_i k_i (K \alpha_i^D - 1) - \beta_i^D}{K (\alpha_i^D)^2} = 0 \quad (15)$$

$$K = \sum_{i=1}^N \frac{1}{\alpha_i^D} + \sum_{j=1}^M \frac{1}{\alpha_j^S}. \quad (16)$$

Therefore, we obtain $\mathbf{L}\boldsymbol{\beta} = \mathbf{c}$, where

$$\boldsymbol{\beta} = [\beta_1^S \dots \beta_M^S \beta_1^D \dots \beta_N^D]^T \quad (17)$$

$$\mathbf{L} = [l_{ji}]_{(N+M) \times (N+M)}$$

$$l_{ii} = \frac{2}{K} - 1 \quad i = 1, 2, \dots, M + N$$

$$l_{ji} = \frac{2 - K \alpha_j^S}{K \alpha_i^S} \quad j, i = 1, 2, \dots, M, \quad i \neq j$$

$$l_{j(i+M)} = \frac{2 - K \alpha_j^S}{K \alpha_i^D} \quad j = 1, \dots, M, \quad i = 1, \dots, N$$

$$l_{(j+M)i} = \frac{2 - K \alpha_j^D}{K \alpha_i^S} \quad j = 1, \dots, N, \quad i = 1, \dots, M$$

$$l_{(j+M)(i+M)} = \frac{2 - K \alpha_j^D}{K \alpha_i^D} \quad j = 1, \dots, N, \quad i = 1, \dots, N, \quad i \neq j$$

$$\mathbf{c}^T = [c_{1i}]_{1 \times (M+N)}$$

$$c_{1i} = 1 + a_j (1 - K \alpha_j^S) \quad i = 1, 2, \dots, M$$

$$c_{1(i+M)} = 1 + r_i k_i (1 - K \alpha_i^D) \quad i = 1, 2, \dots, N \quad (18)$$

TABLE I
THE SPs PARAMETERS

Parameters	α_j^S	a_j	b_j	τ_j
SP 1	0.01-0.1	1.8	0	0.001-0.002
SP 2	0.02-0.2	1.5	0	0.001-0.002
SP 3	0.03-0.3	1.6	0	0.001-0.002

TABLE II
THE USERS PARAMETERS

Parameters	α_i^D	r_i	γ_i	τ_i
User 1	0.03,0.3	10	9-22 db	0.001-0.002
User 2	0.02,0.2	10	9-22 db	0.001-0.002
User 3	0.01,0.1	10	9-22 db	0.001-0.002

And finally, if \mathbf{L} is invertible, the Nash equilibrium point of the game would be:

$$\beta^* = \mathbf{L}^{-1}\mathbf{c}. \quad (19)$$

Therefore, the necessary and sufficient condition for existence of unique Nash equilibrium point is $\det(\mathbf{L}) \neq 0$.

B. Incomplete information case: learning optimal strategy

In a practical cognitive radio environment, users and service providers may only be able to have historical information of price but not the strategies and profits of other users and service providers. Therefore, the players could not reach to the Nash equilibrium point in one shot and the optimal strategies of the players should be learned in a learning game, based on real available information. Since the players are unable to maximize their profits in one shot, they can adjust their strategies based on the marginal profit function. In this adjustment process, the players make their decisions based on local estimation of their marginal payoffs (see, e.g., [16] and [17]). This adjustment mechanism has been called "myopic" by some authors [18], [19]. In a myopic adjustment mechanism based on profit gradient, if the marginal payoff is positive (negative), the player will increase (decrease) its supply quantity for the next time-step. The myopic adjustment equation together with naive expectation (of rivals) and linear speed of adjustment could be written as follows

$$\begin{aligned} \beta_j^S(t+1) &= \beta_j^S(t) \left(1 + \gamma_j^S \frac{\partial \pi_j^S}{\partial s_j}\right), \quad j = 1, 2, \dots, M, \\ \beta_i^D(t+1) &= \beta_i^D(t) \left(1 + \gamma_i^D \frac{\partial \pi_i^D}{\partial d_j}\right), \quad i = 1, 2, \dots, N, \end{aligned} \quad (20)$$

where $0 < \gamma_j^S \leq 1$ and $0 < \gamma_i^D \leq 1$ are constant learning rates of service providers and users, respectively.

Replacing the marginal payoff from (12) and (13) into (20), and using (5), we have

$$\begin{aligned} \beta_j^S(t+1) &= \beta_j^S(t) \\ &\times \left(1 + \gamma_j^S \left(\frac{\lambda(t) + (\lambda(t) - a_j)(1 - \alpha_j^S K) - \beta_j^S(t)}{K(\alpha_j^S)^2}\right)\right), \end{aligned} \quad (21)$$

for $j = 1, 2, \dots, M$, and

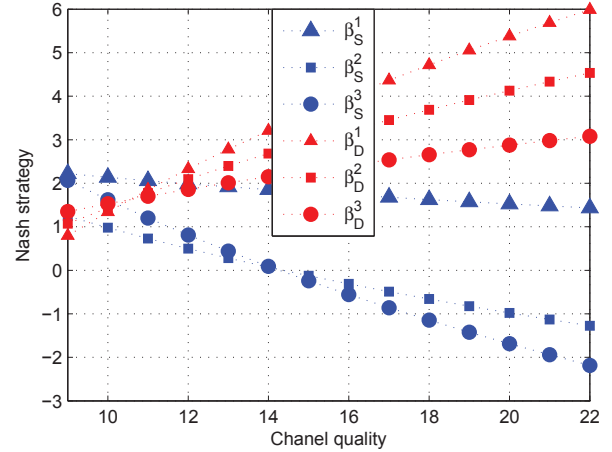


Fig. 3. Nash strategies of SP and users in different channel qualities γ in a network with $\text{BER}_{\text{tar}} = 10^{-4}$.

$$\begin{aligned} \beta_i^D(t+1) &= \beta_i^D(t) \\ &\times \left(1 + \gamma_i^D \left(\frac{\lambda(t) + (\lambda(t) - r_i k_i)(1 - K\alpha_i^D) - \beta_i^D(t)}{K(\alpha_i^D)^2}\right)\right), \end{aligned} \quad (22)$$

for $i = 1, 2, \dots, N$. Therefore, each player needs the value of parameter K in marginal payoff function. Since K is constant, it can be calculated from the following approximate equality

$$\begin{aligned} \frac{\lambda(t) + (\lambda(t) - a_j)(1 - \alpha_j^S K) - \beta_j^S(t)}{K(\alpha_j^S)^2} &= \frac{\partial \pi_j^S(t)}{\partial s_j(t)} \\ &\approx \frac{\pi_j^S(t_2) - \pi_j^S(t_1)}{s_j(t_2) - s_j(t_1)} \triangleq \phi(t_2, t_1), \quad t_1 \leq t \leq t_2 \end{aligned} \quad (23)$$

where t_1 and t_2 are two sample times, and thus, we have

$$K \approx \frac{2\lambda(t) - a_j - \beta_j^S(t)}{(\alpha_j^S)^2 \phi(t_2, t_1) + (\lambda(t) - a_j)\alpha_j^S}. \quad (24)$$

It should be noticed that the accuracy of the estimate can be improved by averaging the obtained parameter over some sample intervals.

It is straightforward to check that the Nash equilibrium point of the game in complete information is also an equilibrium point of the above dynamic game. However, the equilibrium points in a dynamic game can be unstable. In this case, although the players use a rational rule of decision making, they do not reach to the Nash equilibrium point. Hence, it is interesting to examine that in what conditions the Nash equilibrium point is stable, and as a result, the rational behavior of the players in dynamic game converge to Nash equilibrium point.

IV. SIMULATION RESULTS

In this section, we provide numerical results, calculated by assuming channels that are independent Rayleigh distributed with normalized variance.

We consider a cognitive radio environment with three service providers and three users with parameters shown in Table I and II. The target BER for the users is $\text{BER}_{\text{tar}} = 10^{-4}$.

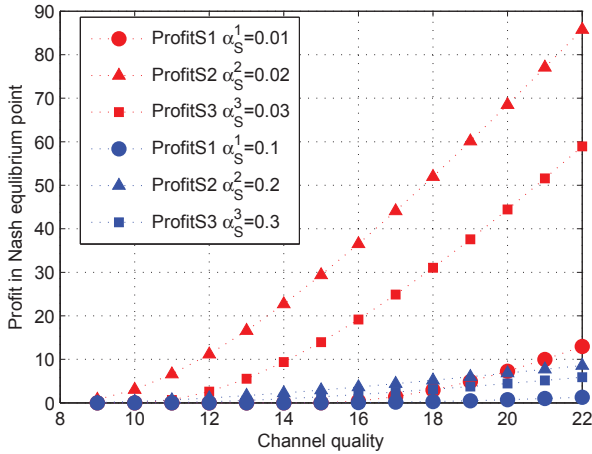


Fig. 4. Profits of SPs in the Nash equilibrium point with different channel qualities by changing the slope of supply function.

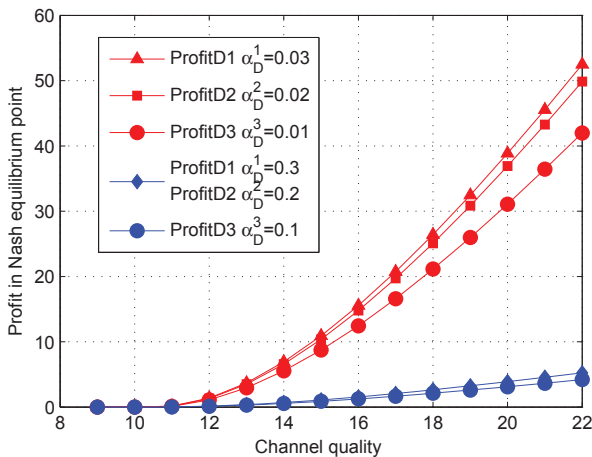


Fig. 5. Profits of users in the Nash equilibrium point with different channel qualities by changing the slope of demand function.

A. Nash Equilibrium Strategies

Fig. 3 shows the Nash equilibrium point of the game by changing the channel quality between 9-22. Note that the channel quality $\gamma_i = \gamma$, $i = 1, \dots, N$, is the SNR at the receiver, as stated in (1). In the Nash equilibrium point, by increasing the channel quality, the strategies of the users become larger, since they are willing to buy more bandwidth due to more satisfaction and the strategies of the service providers become smaller, since they are willing to sell more due to more demand. In addition, it is obvious from matrix \mathbf{L} and vector \mathbf{c} , that if the slopes of supply/demand function increase/decrease with the same order, the Nash equilibrium point is not changed.

Fig. 4 and 5 show the profit of the users and service providers in the Nash equilibrium points, respectively. It is shown that, although the simultaneous changing the slope of supply and demand function has not any effect on Nash equilibrium point, but it has a significant effect on profit. We

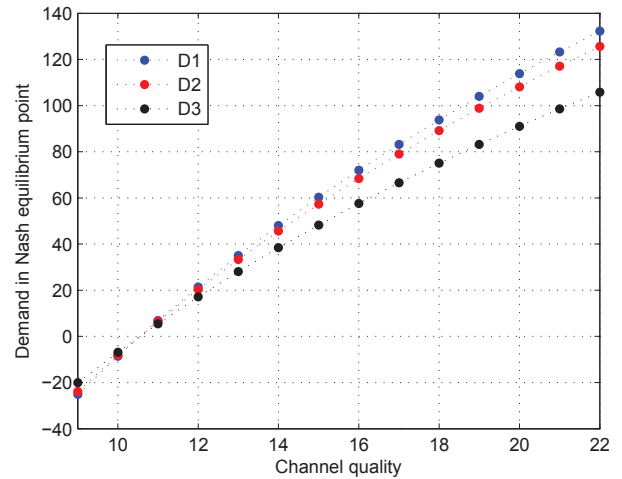


Fig. 6. The cleared bandwidth of users in Nash equilibrium point with different channel qualities when $\alpha_1 = 1.5$.

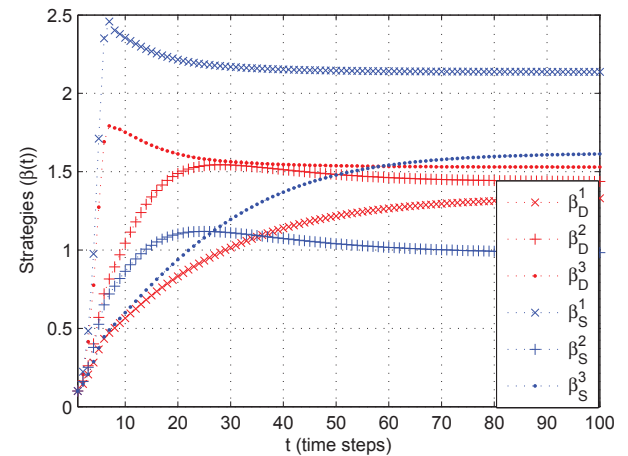


Fig. 7. Strategies of the SPs and users during the learning game and convergence to Nash equilibrium point $\beta^* = [2.137 \ 0.978 \ 1.619 \ 1.340 \ 1.434 \ 1.529]^T$, when learning rate of all players is set to 0.01 and channel quality is 10 db.

can see that the lower slope of the supply/demand functions leads to higher payoff for both service providers and users. The reason is that lower slopes lead to an agreement with larger amount of bandwidth in same price, which is more beneficial for both parties. In addition, it is shown that increasing the channel quality for users result in more profit for both service providers and users. This is a good incentive for service providers to offer bandwidth in high quality channels to amend their performance and satisfy their users, simultaneously.

Moreover the service provider with lowest cost of production ($a_1 = 1.5$) earns more payoff which is an expected result. The same results would be happened with different revenue coefficients for users. Considering users' demand in Nash equilibrium point (Fig. 6), one can realize that that if $r_i k_i - \lambda > 0$ then higher demand results in higher profit and

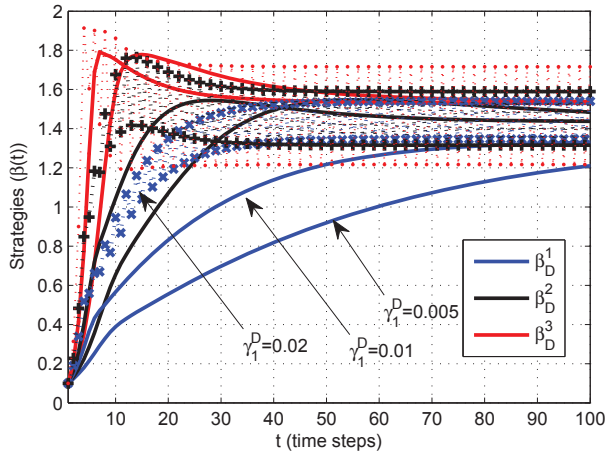


Fig. 8. The effect of different learning rates in stability of the learning game when the learning rate is set to 0.02 and channel quality is 10 db.

vice versa. Moreover, as shown in Fig. 6, in channel qualities of 9 and 10, since $r_i k_i - \lambda \leq 0$, the users get zero profit.

B. Learning game

Fig. 7 shows convergence of the players' strategies to Nash equilibrium point in the case that learning rate of all players is set to 0.01 and channel quality is 10 db. After 100 round of the game, the strategies of service providers converge to (2.1372 0.9827 1.6134) and the strategies of users converge to (1.3307 1.4373 1.5296), respectively. The Nash equilibrium point of the game is calculated as $\beta^* = [2.137 \ 0.978 \ 1.619 \ 1.340 \ 1.434 \ 1.529]^T$. Therefore, the players are able to learn the Nash strategy by gradient dynamics with incomplete information of the game. Fig. 7 shows the effect of different learning rate on convergence to Nash equilibrium point. For clarity of figure only the strategies of users are shown with learning rates of 0.005, 0.01 and 0.02. It can be seen that smaller learning rates lead to slower convergence to equilibrium point. Therefore, it is rational to increase learning rate for increasing the performance. However, increasing learning rate from a specific threshold may lead to instability of the system. As it can be seen from Fig. 8, when the learning rate is set to 0.02, the system is in the limit of stability and instability (resonance). Therefore, the learning rates larger than 0.02 result in instability of the strategies.

V. CONCLUSION

In this paper, we proposed a new scheme based on supply and demand curves for SPs and users, instead of bidding a constant value of price or bandwidth by SPs and users as their bids. This results in the possibility of reaching an agreement in wider ranges of price and bandwidth. It was shown that the game has a unique explicit Nash equilibrium point under a simple condition. In addition, a gradient based learning method was proposed for decision making of the SPs and users in incomplete information case, when they only have the historical information of the price. In fact, the equilibrium

point of the learning dynamics was the Nash equilibrium point of the game. It was shown that by choosing small learning rates, the learning game converges to the Nash equilibrium point as well. Simulation results showed that increasing the channel quality for users result in more profit for both service providers and users. This is a good incentive for service providers to offer bandwidth in high quality channels to amend their performance and satisfy their users, simultaneously.

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